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ANALYSIS OF SPRINGBACK
IN BENDING OF METALS

A THESIS

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the Faculty of the Graduate Division

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IN BENDING OF METALS

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LIST OF SYMBOLS

SYMBOL	DEFINITION
A-A and B-B	Reference radial cross-sections
c	Incremental fiber length, before bending
c_1	Incremental fiber length, after bending
c_2	Incremental fiber length, after springback
d_1	Distance from inner fiber to neutral surface of springback
d_2	Distance from outer fiber to neutral surface of springback
e	Conventional, or engineering, strain
e_1	Strain after bending
e_2	Strain after springback
e_a and e_a'	Strain parameters of straight-line stress-strain curve
e_m	Minimum fiber strain in bending
e_M	Maximum fiber strain in bending
e_p	Strain at the proportional limit
e_u	Ultimate elongation in tension, two inch gage length
E	Modulus of elasticity
E_1, E_1', E_2, and E_2'	Slope parameters of straight-line stress-strain curve
f	General function expressing stress in simple tension-compression test
I_z	Centroidal moment of inertia of cross-section about z-axis

SYMBOL	DEFINITION
K and K'	Constants
L-L	Traces of the neutral surface of springback
m	Constant
M	Applied moment acting upon member
n	Constant
N-N	Traces of the neutral surface in bending
r	Bent radius of curvature of neutral surface in bending
r'	Final radius of curvature of neutral surface of springback
R_d	Bent radius of curvature of innermost fiber
R_p	Final radius of curvature of innermost fiber
s	Conventional, or engineering, stress
s_1	Stress after bending
s_2	Stress after springback
s_a and s_a'	Stress parameters of straight-line stress-strain curve
s_m	Minimum fiber stress in bending
s_M	Maximum fiber stress in bending
s_o	Constant
s_u	Tensile strength of material
s_y	Yield strength in tension
s_y'	Yield strength in compression
t	Thickness of member
t_1	Distance from inner fiber to neutral surface in bending
t_2	Distance from outer fiber to neutral surface in bending

SYMBOL	DEFINITION
w	Width of member
x	Axial coordinate
y	Radial coordinate, origin at neutral surface in bending
y'	Radial coordinate, origin at neutral surface of springback
y_a and y_a'	Distance parameters of straight-line stress-strain curve
z	Transverse coordinate
Θ	Angle between two cross-sections after bending
ϕ	Angle between same two cross-sections after springback
ϕ/Θ	Springback ratio
arctan	Inverse tangent of
exp	Exponential function of, as in exp(a + b) for e^{a+b}, where e is the base of natural logarithms
ln	Natural logarithm of

SUMMARY

One serious difficulty in the precise forming of metal members arises from the phenomenon termed "springback." Springback, an elastic phenomenon, is the tendency of a bent member to return to its original shape upon removal of the forming forces.

The design of forming tools to produce accurate bends in metal parts is largely a trial-and-error process because of this springback phenomenon. Although several analyses of springback have been published, they generally either are not sufficiently accurate or are very time-consuming when applied to actual conditions. In this paper, it was desired to develop a simplified mathematical analysis of springback and to investigate the applicability of two different analytical approximations of the stress-strain curve to this analysis.

Papers presenting analyses of springback and experimental springback data and some investigations of bending of metals in the plastic range were listed and briefly discussed.

A general expression for springback was derived for an initially straight, uniform member undergoing pure bending, using the major assumptions of the classic beam

theory. The general expression for springback was applied to the case of a rectangular cross-section.

A straight-line-segment and an exponential type approximation of the stress-strain curve were chosen, and each was applied to the springback relation developed.

The two approximations derived were applied to three materials for which both stress-strain curves and springback data were available--2024-T, 6061-T6, and 2024-0 aluminum alloy sheets. Comparisons were then made between the approximations derived, available test data, and three previously published theories. It was found that both approximations were very nearly equal except for large bend radii, where the straight-line segment is not valid. It was also found that the approximations presented were more accurate than two of the other analyses considered. They were only slightly less accurate than the other analysis discussed, which was developed for 2024-T Alclad sheet.

It seems best to use the straight-line approximation where applicable, since this relation requires less calculation than the exponential approximation. However, for large bend radii or where complete stress-strain curves are not available, the exponential approximation is recommended.

CHAPTER I

INTRODUCTION

The Problem of Springback in Metal Forming.--The fabrication of many commodities and structures in this industrial age requires the bending of metal parts to a closely specified shape or contour. This is particularly true in aircraft manufacture, since the majority of the structural parts of a modern aircraft are sheet metal members which must be formed with a high degree of accuracy.

Bending of metal members, while seemingly a simple operation, is subject to many difficulties in actual practice. One of the greatest of these difficulties lies in a phenomenon of elastic materials termed springback. Simply stated, springback is the tendency of a bent member to return to its original shape upon removal of the forming forces. The immediate consequence of springback is that the contour of a member which has been bent is not the contour produced by the forming equipment, but is somewhere intermediate to the formed contour and the original contour of the member. In general, the amount of springback varies with the shape of the member, the material of the member, and the method of forming.

The usual practice in the fabrication of bent parts is for the tool designer, relying upon his past experience with metals and metal-forming equipment, to estimate as closely as possible the amount of springback for each particular new part and method of forming. The tool designer then attempts to compensate for springback in some manner, such as overbending, in designing tools for the new part. If his estimate of the amount of springback is excessively in error, such that the final part is not within manufacturing tolerances, each part must be finish-formed by hand or the forming tools have to be corrected. The correction of forming tools remains a trial-and-error process in most cases. Since correction of a part contour by hand is an expensive and time-consuming process, and since tool making and reworking is also very expensive, it would be advantageous to be able to predict accurately the amount of springback for any given part and method of forming.

Several attempts have been made to analyze the phenomenon of springback in bending. In general, the relations which have been found to express the amount of springback are quite complex and therefore difficult to evaluate in any particular case. The more complete and thorough of these analyses involve trial-and-error numerical solutions, which are laborious and time-consuming. Other analyses, which enable springback to be found by direct calculation, include untenable assumptions in their derivations and yield results which do not usually compare with actual values with sufficient accuracy for precision members, as in aircraft fabrication or high-speed turbo-machinery.

Purpose and Scope of Investigation.--The purpose of this work is to present a simplified mathematical analysis of the bending of metals in the plastic range and the phenomenon of springback. For simplicity, the assumptions of the classical beam theory of mechanics are used, and only the case of pure bending is considered. Two different analytical approximations to the stress-strain curve of the simple tension-compression test are used in applying the results of the analysis to several commonly used materials.

Since considerable experimental data on springback exist in the literature for straight bends on rectangular cross-sections, no experimental work was performed in connection with this investigation.

Comparison is made between several previously published papers on springback and the analysis presented here, and between the results of the various theories. Comparison is also made between the two stress-strain approximations used in the present work and between experimental data drawn from the literature.

Review of the Literature.--Some of the investigations of the bending of metals in the plastic range which have been published in recent years are those of Cozzone (1)*, Marin and Cotterman (2), Osgood (3) (4) (5), Wang (6), Williams (7), Swift (8), Sachs and Lubahn (9), Sachs, Lubahn, and Taub (10) (11), and Lubahn and Sachs (12).

The first eight of these papers make use of the basic assumptions of the classic beam theory, neglecting stresses other than the normal stress in the longitudinal direction of the bent member and neglecting the effect of any change of cross-sectional shape during bending on stresses within the member. Cozzone and Wang analyzed bending for the extreme fiber stresses and for the applied bending moment for given curvatures. Marin and Cotterman, Osgood, and Williams analyzed bending for the extreme fiber strains and applied bending moment for given curvatures. Marin and Cotterman and Osgood also derived relations for small deflections in bending. Swift analyzed pure bending with a superimposed constant longitudinal tension for the thinning of metal under simple and reversed bending. Cozzone replaced the tension and compression stress-strain test curve by a trapezoidal diagram (see Fig. 6 (e)), equal in tension and compression, and assumed that the neutral axis in bending passes through the centroid of the member. Marin and Cotterman replaced the stress-strain curve by a straight line

---

*Figures in parentheses refer to the Bibliography at the end of this paper.

segment approximation (Fig. 6 (f)), and obtained excellent agreement of their theoretical results with experimental data. Osgood assumed a stress-strain curve of the form of Figure 6 (d), and one of the form* $e = (s/E) + K(s/E)^n$, where K and n are empirical constants. Swift assumed a stress-strain curve of the form of Figure 6 (e). Wang and Williams recommended graphical integration of the actual stress-strain curve for a solution, but Wang also presented an analytical representation of the curve which appears to give good results as compared with the actual test curve.

Sachs and Lubahn (9) made a rather thorough analysis of pure bending under conditions of plane strain (i.e., bending of an infinitely wide sheet), and Sachs, Lubahn, and Taub (10) performed a similar analysis of pure bending under conditions of plane stress (i.e., edgewise bending of an infinitely thin sheet). Lubahn and Sachs (12) presented the material of these two papers in condensed form. Both these analyses assumed a perfectly plastic material (Fig. 6 (c)) and the validity of the distortion-energy theory and considered only rectangular members. They used a graphical-numerical method of successive approximations to determine relations between the inner radius of curvature of the member and the height of the member, the position of the neutral axis, and

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*For the definition of symbols used, see the List of Symbols, p. vi.

the lateral, tangential, and radial strain distributions. Sachs, Lubahn, and Taub (11) conducted an experimental investigation of bending to verify certain of the results of the above analyses and to attempt to isolate certain secondary variables in bending.

Investigations and analyses of springback in the bending of metals have been published by Strasser (13), Hazlett and Schroeder (14), Lee (15), Sturm and Fletcher (16), Schroeder (17), Oestreich (18), Brown, Binder, and Franks (19) (20), Dorn, Jelinek, and Ballaseyus (21), Schroeder (22), and Gardiner (23).

Strasser listed some of the determining factors of springback in die forming and outlined several methods for reducing springback. Hazlett and Schroeder gave a qualitative analysis of tests made to isolate the variables affecting springback of rubber-formed flanges. Lee proposed the empirical relation $(\theta - \varnothing) = K\ t^n\ Rd^m$, where K, n, and m are empirical constants for any material, and gave values for tests made on 2024-0 and 2024-T Alclad sheets.

All the theoretical analyses presented are based upon the basic assumptions of the classic beam theory of pure bending. Sturm and Fletcher and Oestreich used a numerical method of successive approximations, dividing the cross-section into layers over which stress is assumed constant, as given by the actual stress-strain test curve, and assuming values of the displacement of the neutral axis in bending. Sturm and Fletcher assumed that the neutral axis remains at the neutral axis in bending during springback, while Oestreich assumed that the neutral axis is at the centroid during springback, but displaced during bending. Schroeder (17) presented a graphical-numerical method for finding springback, using the actual stress-strain curve and assuming that the neutral axis passes through the centroid of the cross-section at all times. Schroeder (17) also presented, without derivation, formulas for calculating directly the springback of aluminum alloy and Alclad sheets. Brown _et al_. used the analysis of Schroeder (17) to present alignment charts to determine the final formed angle of a bent part for various tempers of 18-8 and 17-7 stainless steels. Dorn _et al_. presented a graphical-numerical method for determining springback, using the actual stress-strain curve and making no assumptions as to the location of the neutral axis. Gardiner assumed an elasto-plastic material (Fig. 6 (d)) with equal properties in tension and compression and assumed the neutral axis remains at the centroid of the cross-section in developing a generalized method for finding springback. Schroeder (22) investigated pure bending with a superimposed constant longitudinal tension to find the springback in the plane of bending and the distortion perpendicular to the plane of bending. He assumed a plastic material with linear strain-hardening (Fig. 6 (e)), and assumed that the neutral axis remains at the centroid of the cross-section.

Considerable data on springback have been presented in the literature, both results of tests on various materials and data based on industrial forming experience. Data for aluminum alloys are presented by Chapman, Hazlett, and Schroeder (24), Schroeder and Hazlett (25), Lee (15), Schroeder (17), Sachs, Doll, Seybolt, Meinel, and Clark (26), and Sachs and Espey (27). Sachs (28) presents data for aluminum alloys and various steels. Dorn, Jelinek, and Ballaseyus (21) present detailed test results on six magnesium alloys at room and elevated temperatures. Gardiner (23) gives test data for a variety of materials.

CHAPTER II

ANALYSIS OF BENDING AND SPRINGBACK

Assumptions.--Let there be considered a member subjected to pure bending in an axial plane which contains one of the two principal axes of the cross-section.

The following assumptions shall be made, in accordance with the classic beam theory of bending:

(1) The conditions of stress and strain will be the same on every radial cross-section. This assumption implies that every plane cross-section normal to the axis of the member remains plane and normal to the axis during (and after) bending. Such an assumption is commonly accepted and has been experimentally justified repeatedly for beams or plates without shear loading, as is the case in this analysis. It is to be recognized that St. Venant's Principle applies here also and that this assumption is not strictly valid near the ends of the bend due to local variations of the strain distribution.

(2) The same stress-strain relation for each element (or fiber) of the material holds in bending as in simple tension and compression.

(3) Each longitudinal fiber of the material behaves

as if it were independent of every other longitudinal fiber. This assumption implies that the behavior of every longitudinal fiber is not affected by lateral forces or by shearing stresses between the fibers. While not strictly correct, this assumption is a basic assumption of the classic beam theory and gives a sufficiently close approximation to actual conditions.

(4) The moduli of elasticity in tension and compression are equal.

In addition to the foregoing assumptions, the following simplifying conditions shall be imposed:

(5) The material is homogeneous, isotropic, and is free from initial residual stress.

(6) The member has a uniform cross-section.

(7) The member is initially straight.

(8) During unloading, each fiber follows Hooke's Law.

Bending Considerations.--Under the specified loading, producing pure bending, the bending moment acting upon each radial cross-section is constant along the length of the member. According to Assumptions 5 and 6, each portion of the member bounded by equally-spaced radial planes is identical. Therefore, all such portions deform equally under a given moment. According to Assumption 1, during bending each plane cross-section normal to the axis of the member remains plane and normal to the axis of the member. Thus it follows that each such cross-section lies on the radius of curvature of the bent member and that the member is deflected into a circular arc.

In each radial cross-section of the deflected member there is a line, the neutral axis of bending, which contains all fibers in a state of zero strain and which is perpendicular to the plane of bending. The summation of all such lines is the neutral surface in bending, traces of which are denoted N-N in Figure 1. Since each fiber in the neutral surface is in a state of zero strain and since, by Assumption 3, each fiber is independent of all others, the stress acting on each fiber in the neutral surface is zero.

Figure 1 shows an element of the member deflected so that the neutral surface in bending describes an arc of radius r. Cross-sections A-A and B-B, normal to the neutral surface, subtend the angle θ. The length of any fiber in the neutral surface is c. By Assumption 7, since such a fiber is unstrained, the original length of all fibers subtended by the angle θ is also c.

From Figure 1

$$c = r\theta \tag{1}$$

Also, the length of any fiber between A-A and B-B, denoted by c_1, is

$$c_1 = (r + y)\theta \tag{2}$$

Thus the conventional strain, e_1, of any longitudinal fiber

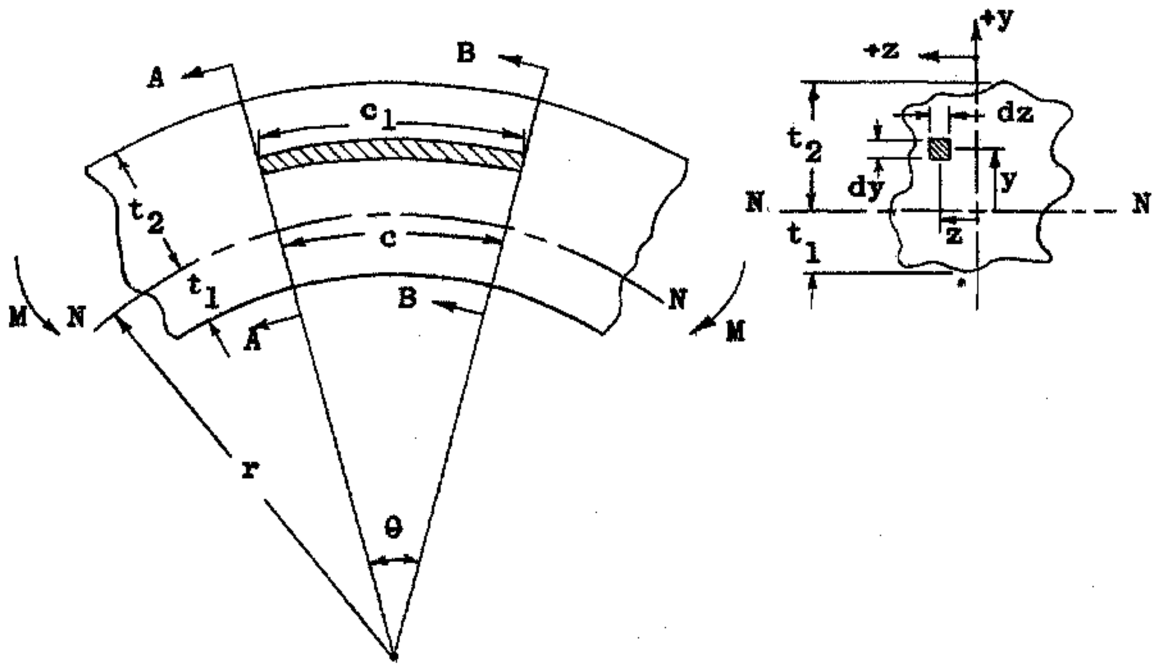

Figure 1. Element of Member After Bending

after bending is, by definition

$$e_1 = \frac{c_1}{c} - 1 = \frac{(r + y)\theta}{r\theta} - 1$$

or

$$e_1 = y/r \tag{3}$$

From Equation (3) it is seen that the maximum longitudinal strain, e_M, and the minimum longitudinal strain, e_m, of the bent member are given by

$$e_M = t_2/r \text{ and } e_m = - t_1/r \tag{4}$$

For static equilibrium of the deformed member, the summation of the stress forces across any cross-section must be zero at all times. That is

$$\iint s_1 dydz = 0 \tag{5}$$

across any cross-section, where s_1 is the stress acting at any point in the bent member.

Also for static equilibrium, the summation of the moments of all stress forces acting on any cross-section must be equal to the moment acting on that cross-section. Since the loading is restricted to pure bending only, the acting moment, M, is constant along the length of the member. Thus,

$$\iint s_1 y\,dy\,dz = M \tag{6}$$

across any cross-section.

By Assumption 2, any fiber which is loaded such that its deformation is always increasing in magnitude will exhibit the same relation between the strain of the fiber and the stress within the fiber as is given by the stress versus strain curve of the simple tension-compression test. The tension-compression test indicates that the stress within a deformed fiber is a single-valued function of the strain which the fiber undergoes. This relation may be represented by

$$s = f(e) \tag{7}$$

where the indicated function defines the stress-strain curve of the simple tension-compression test. In Equations (5) and (6), the stress is given by Equation (7) as

$$s_1 = f(e_1) \tag{8}$$

since the deformation of the member is achieved by bending in one direction only.

Springback Considerations.--A member loaded in tension (or compression) will exhibit a stress-strain relation like that given by the curve OAB in Figure 2. If such a member is loaded to point A and the load is then removed, the resulting stress-strain relation will be given by a curve

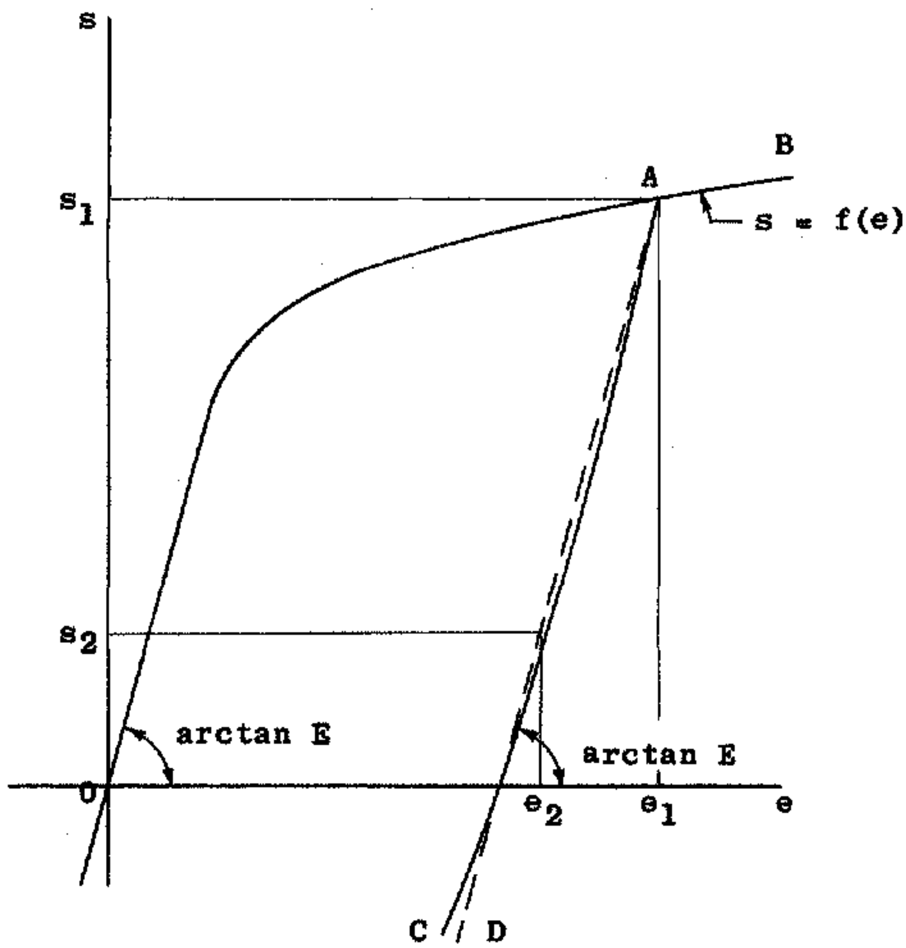

Figure 2. Stress-Strain Curve for Unloaded Fiber

such as AC. Various experimenters have demonstrated for many metals that the curve AC is essentially straight for complete unloading and even for considerable reversed loading and that the slope of the curve AC is approximately equal to the modulus of elasticity, E. Hence, it is here assumed (Assumption 8) that the path followed during unloading is given by the dotted line AD of Figure 2.

Therefore, by Assumptions 2 and 8, it is seen from Figure 2 that on unloading a fiber which has been deformed to a strain e_1 at the stress s_1, the following relation exists:

$$s_2 = s_1 - E(e_1 - e_2) \quad \text{where} \begin{cases} e_1 \geqslant e_2 \text{ if } e_1 > 0 \\ e_1 \leqslant e_2 \text{ if } e_1 < 0 \end{cases} \tag{9}$$

It is clear that if a fiber which has been deformed to a strain e_1 by a stress s_1 is further deformed in the same direction to a strain e_2, the final stress acting on the fiber is still given by the stress-strain curve. That is

$$s_2 = f(e_2) \quad \text{where} \begin{cases} e_1 \leqslant e_2 \text{ if } e_1 > 0 \\ e_1 \geqslant e_2 \text{ if } e_1 < 0 \end{cases} \tag{10}$$

Now consider that the member has been deformed into an arc such that the neutral surface in bending has achieved a radius r, as previously discussed. If the loading on the member is now removed, the bending moment M acting on the member decreases to zero throughout the length of the member, and the member will spring back to some smaller curvature. The same arguments previously used assure that the final form of the member is a circular arc.

Exactly as occurred in bending, in each radial cross-section of the member after springback there is a line, the neutral axis of springback, which is perpendicular to the plane of bending and which contains all fibers whose state of strain after springback is the same as before springback. The summation of all such lines is the neutral surface of springback, traces of which are denoted L-L in Figure 3.

In Figure 3 the surface N-N is the neutral surface in bending, r is the radius which this surface obtains in bending, and cross-sections A-A and B-B subtend the angle θ after bending. During springback, the cross-sections A-A and B-B effectively rotate about their neutral axes of springback to the positions denoted by A'-A' and B'-B' respectively. In their final formed positions, after springback, these cross-sections subtend the angle ϕ, and the neutral surface of springback has a radius of r'.

Before bending, the length of any fiber between cross-sections A-A and B-B, denoted by c, is given by Equation (1) as

$$c = r\theta \tag{1}$$

After bending and before springback, the length of any fiber between cross-sections A-A and B-B, denoted by c_1, is

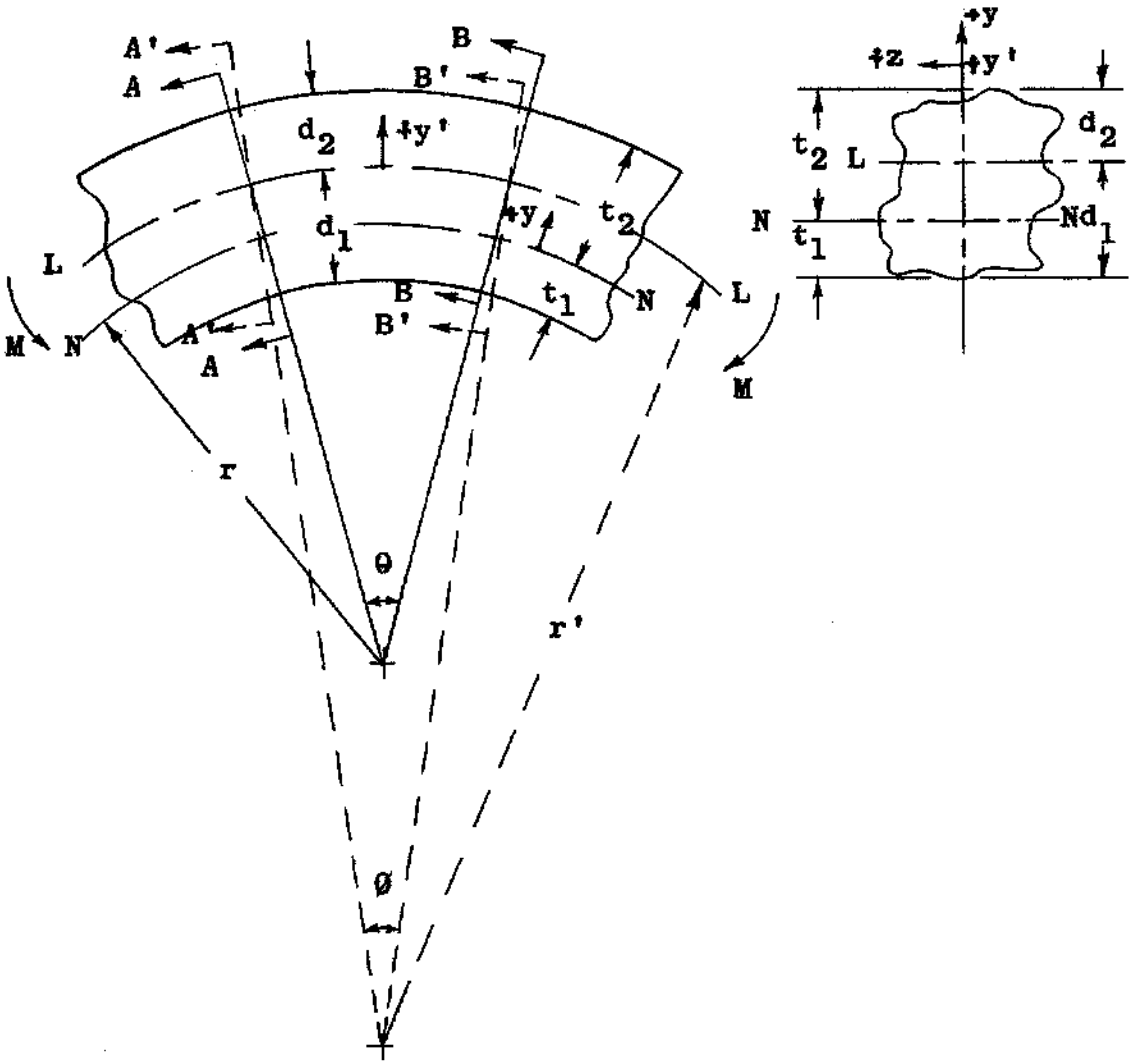

Figure 3. Element of Member Before and After Springback

given by Equation (2) as

$$c_1 = (r + y)\theta \tag{2}$$

After springback, the length of any fiber between the same cross-sections (now in positions A'-A' and B'-B' respectively), denoted by c_2, is found from Figure 3 to be

$$c_2 = (r' + y')\phi \tag{11}$$

Also from the figure, it is seen that for any particular fiber

$$d_1 + y' = t_1 + y \tag{12}$$

Thus, from Equations (2) and (12)

$$c_1 = (r + y' + d_1 - t_1)\theta \tag{13}$$

From the definition, the strain of any fiber before springback, denoted by e_1, is

$$e_1 = \frac{c_1}{c} - 1 = \frac{1}{r}(y' + d_1 - t_1) \tag{14}$$

Similarly, the strain of any fiber after springback, denoted by e_2, is given by

$$e_2 = \frac{1}{r}(r' + y')\frac{\phi}{\theta} - 1 \tag{15}$$

Since any fiber on the neutral surface of springback has the same strain after springback as before springback, for such a fiber $e_1 = e_2$. Thus, substituting Equations (14) and (15) for such a fiber (where y' is zero) yields

$$d_1 - t_1 = r'\frac{\phi}{\theta} - r \tag{16}$$

Hence, substituting into Equation (14) yields

$$e_1 = \frac{1}{r}(y' + r'\frac{\phi}{\theta}) - 1 \tag{17}$$

From Equations (15) and (17) one obtains

$$e_1 - e_2 = \frac{1}{r}(1 - \frac{\phi}{\theta})y' \tag{18}$$

The quantity ϕ/θ presents a measure of the springback occurring between the cross-sections A-A and B-B, being the ratio between the final, or formed, angle to the bending angle between these cross-sections. Since the member being considered is uniform along its length and the loading on the member is also constant along the length, it follows that the ratio ϕ/θ is a constant without regard to the size of the increment of angle taken as θ.

If no springback should occur in the member after unloading, then ϕ would be equal to θ and the curvature of the member would not change. However, if springback does occur, then ϕ will be smaller than θ. Furthermore, it is known from experience that after bending an originally straight member to some curvature and releasing the member, the curvature will decrease, perhaps even to zero, but the member will not acquire a resultant curvature in the opposite direction from which it was bent. Therefore, the quantity ϕ/θ is always positive and has an absolute maximum value of one.

Hence, since r is a positive quantity, according to Equation (18)

$$
\begin{aligned}
e_1 &> e_2 \quad \text{where} \quad y' > 0, \\
e_1 &= e_2 \quad \text{where} \quad y' = 0, \text{ and} \\
e_1 &< e_2 \quad \text{where} \quad y' < 0,
\end{aligned}
\tag{19}
$$

From Equation (14) it is seen that

$$
\begin{aligned}
e_1 &> 0 \quad \text{where } y' > t_1 - d_1, \\
e_1 &= 0 \quad \text{where } y' = t_1 - d_1, \text{ and} \\
e_1 &< 0 \quad \text{where } y' < t_1 - d_1.
\end{aligned}
\tag{20}
$$

Using the inequalities of Equations (19) and (20), the stress on any fiber after springback may be found from Equations (9) and (10). Thus,

$$
s_2 = s_1 - E(e_1 - e_2), \tag{21}
$$

$$
\text{where}\begin{cases} y' \geq 0 \text{ or } y' \leq t_1 - d_1, \text{ if } t_1 - d_1 \leq 0 \\ y' \leq 0 \text{ or } y' \geq t_1 - d_1, \text{ if } t_1 - d_1 \geq 0 \end{cases}
$$

and

$$s_2 = f(e_2), \tag{22}$$

$$\text{where} \begin{cases} t_1 - d_1 \leqslant y' \leqslant 0, \text{ if } t_1 - d_1 \leqslant 0 \\ 0 \leqslant y' \leqslant t_1 - d_1, \text{ if } t_1 - d_1 \geqslant 0 \end{cases}$$

Through the use of Equations (8), (14), (15), (16), and (18), Equations (21) and (22) become, in terms of y',

$$s_2 = f\left[\frac{1}{r}(y' + d_1 - t_1)\right] - \frac{E}{r}(1 - \frac{\phi}{\theta})y' \tag{23}$$

$$\text{where} \begin{cases} y' \geqslant 0 \text{ or } y' \leqslant t_1 - d_1, \text{ if } t_1 - d_1 \leqslant 0 \\ y' \leqslant 0 \text{ or } y' \geqslant t_1 - d_1, \text{ if } t_1 - d_1 \geqslant 0 \end{cases}$$

and

$$s_2 = f\left[\frac{1}{r}(y'\frac{\phi}{\theta} + d_1 - t_1)\right], \tag{24}$$

$$\text{where} \begin{cases} t_1 - d_1 \leqslant y' \leqslant 0, \text{ if } t_1 - d_1 \leqslant 0 \\ 0 \leqslant y' \leqslant t_1 - d_1, \text{ if } t_1 - d_1 \geqslant 0 \end{cases}$$

Where y' = 0, Equations (23) and (24) both give

$$s_2 = f\left[(d_1 - t_1)/r\right] \tag{25}$$

Where $y' = t_1 - d_1$, Equation (23) gives

$$s_2 = \frac{E}{r}(1 - \frac{\phi}{\theta})(d_1 - t_1) \tag{26}$$

and Equation (24) gives

$$s_2 = f\left[\frac{1}{r}(1 - \frac{\phi}{\theta})(d_1 - t_1)\right] \tag{27}$$

The occurrence of two expressions for the stress after springback at $y' = t_1 - d_1$ stems from the straight-line approximation made in Equation (9). Consideration of the two stress expressions of Equations (9) and (10) shows that if the strain at the neutral axis of springback does not exceed the strain at the proportional limit of the material, then both equations are identical where y' is between 0 and $(t_1 - d_1)$. If this is true, then Equations (9) and (23) are valid for all y'.

The working assumption will now be made that the strain limitation above is satisfied. That is, at $y' = 0$

$$\left|e_2\right| = \left|e_1\right| = \left|(d_1 - t_1)/r\right| \leq \left|e_p\right| \tag{28}$$

where e_p is the strain at the proportional limit. If $(d_1 - t_1)$ is positive, e_p is at the proportional limit in tension; if $(d_1 - t_1)$ is negative, e_p is at the proportional limit in compression.

For static equilibrium after springback, the summation of stress forces across any cross-section must be zero, or

$$\iint s_2 dy'dz = 0 \tag{29}$$

Also for static equilibrium after springback, the summation of the moments of all stress forces acting on any cross-section must be equal to the moment acting on that cross-section. Since after springback the member is unloaded and the moment acting on any cross-section is zero,

$$\iint s_2 y' dy' dz = 0 \tag{30}$$

integrated over any cross-section.

Applying Equation (9) to Equation (29) gives

$$\iint s_2 dy' dz = \iint s_1 dy' dz - E \iint (e_1 - e_2) dy' dz = 0 \tag{31}$$

From Equation (12) it is seen that

$$dy' = dy \tag{32}$$

Applying Equations (5) and (32) and substituting Equation (18), there results

$$\frac{E}{r}(1 - \frac{\phi}{\theta}) \iint y' dy' dz = 0 \tag{33}$$

If the member is bent, then r is finite, and if springback occurs, then the ratio ϕ/θ is less than one. Therefore, except for the trivial cases of no bending and no springback, Equation (33) reduces to

$$\iint y' dy' dz = 0 \tag{34}$$

Since Equation (34) states that the moment of the cross-sectional area of the member about the neutral axis of springback is zero, then the neutral axis of springback passes through the centroid of the cross-section. Further, since the neutral axis of springback is perpendicular to the plane containing one principal axis of the cross-section, the neutral axis of springback lies along the other principal axis of the cross-section.

Applying Equation (9) to Equation (30) gives

$$\iint s_2 y'dy'dz = \iint s_1 y'dy'dz - E\iint (e_1 - e_2)y'dy'dz = 0 \tag{35}$$

Substituting Equations (12) and (32) yields

$$(t_I - d_I)\iint s_1 dydz + \iint s_1 ydydz - E\iint (e_1 - e_2)y'dy'dz = 0 \tag{36}$$

Applying Equations (5) and (6) and substituting Equation (18), this reduces to

$$\frac{E}{r}\left(1 - \frac{\phi}{\theta}\right)\iint y'^2 dy'dz = M \tag{37}$$

The integral in Equation (37) will be recognized as the moment of inertia of the cross-sectional area about the neutral axis of springback, or about the principal axis. Denoting this integral by I_z and solving Equation (37) for the springback ratio, it is found that

$$\frac{\phi}{\theta} = 1 - \frac{Mr}{EI_z} \tag{38}$$

Inspection of Equations (3), (6), and (8) reveals that the bending moment M is a function of the bent radius of curvature r, the shape of the cross-section of the member, and the shape of the stress-strain curve of the material of the member. E is a function of the material used and I_z is a function of the shape of the cross-section. Therefore, it has been shown that for the case under consideration, the springback ratio ϕ/θ is a function of the bent radius of curvature, the shape of the cross-section of the member, and the molulus of elasticity and shape of the stress-strain curve of the material of the member.

Application to Rectangular Section.--The application of the previously developed theory to members having rectangular cross-sections results in some simplification. Since the cross-section is rectangular, the principal axes are normal to and bisect the sides of the rectangle and intersect at the centroid, or the center of the cross-section. The width of the member, or dimension normal to the plane of bending, shall be denoted as w, and the thickness, or radial dimension in the plane of bending, shall be denoted as t, as indicated in Figure 4. Since no strains or stresses normal to the longitudinal fibers of the members have been considered in this analysis, it is assumed that these dimensions do not change during or after bending. Since the cross-section is rectangular, the dimensions locating the neutral axis of springback, d_1 and d_2, are equal. That is

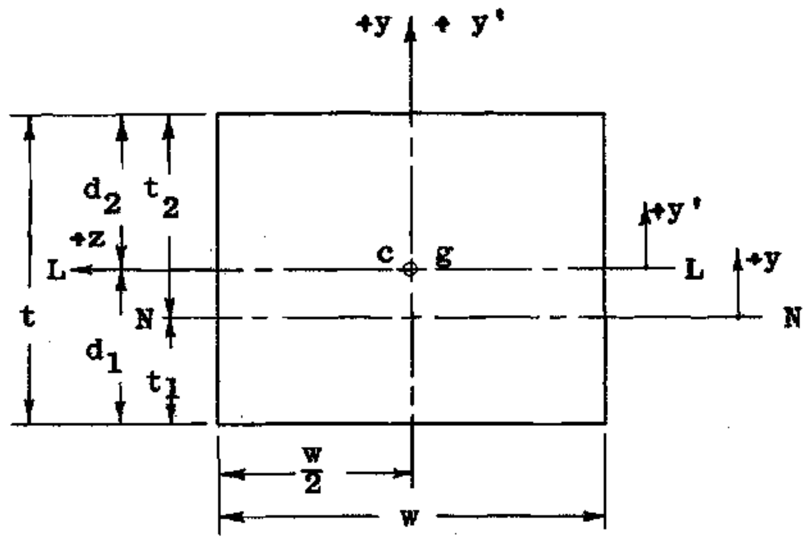

Figure 4. Cross-Section of Rectangular Section

$$d_1 = d_2 = t/2 \tag{39}$$

Writing Equation (5) and inserting the limits of integration as indicated by Figure 4 gives

$$\int_{-w/2}^{w/2}\int_{-t_1}^{t_2} s_1 dydz = 0 \tag{40}$$

Thus, integrating once and simplifying gives

$$\int_{-t_1}^{t_2} s_1 dy = 0 \tag{41}$$

Similarly, Equation (6) as applied to a rectangular cross-section becomes

$$w\int_{-t_1}^{t_2} s_1 ydy = M \tag{42}$$

For a rectangular section, Equation (38) may be written

$$\frac{\phi}{\theta} = 1 - \frac{12Mr}{Ewt^3} \tag{43}$$

Solution Using Straight-Line Approximation of Stress-Strain Curve.--An approximation to the stress-strain curve as determined from the simple tension-compression test, consisting of four straight-line segments, is shown in Figure 5. Such an approximation was assumed by Joseph Marin and F. D.

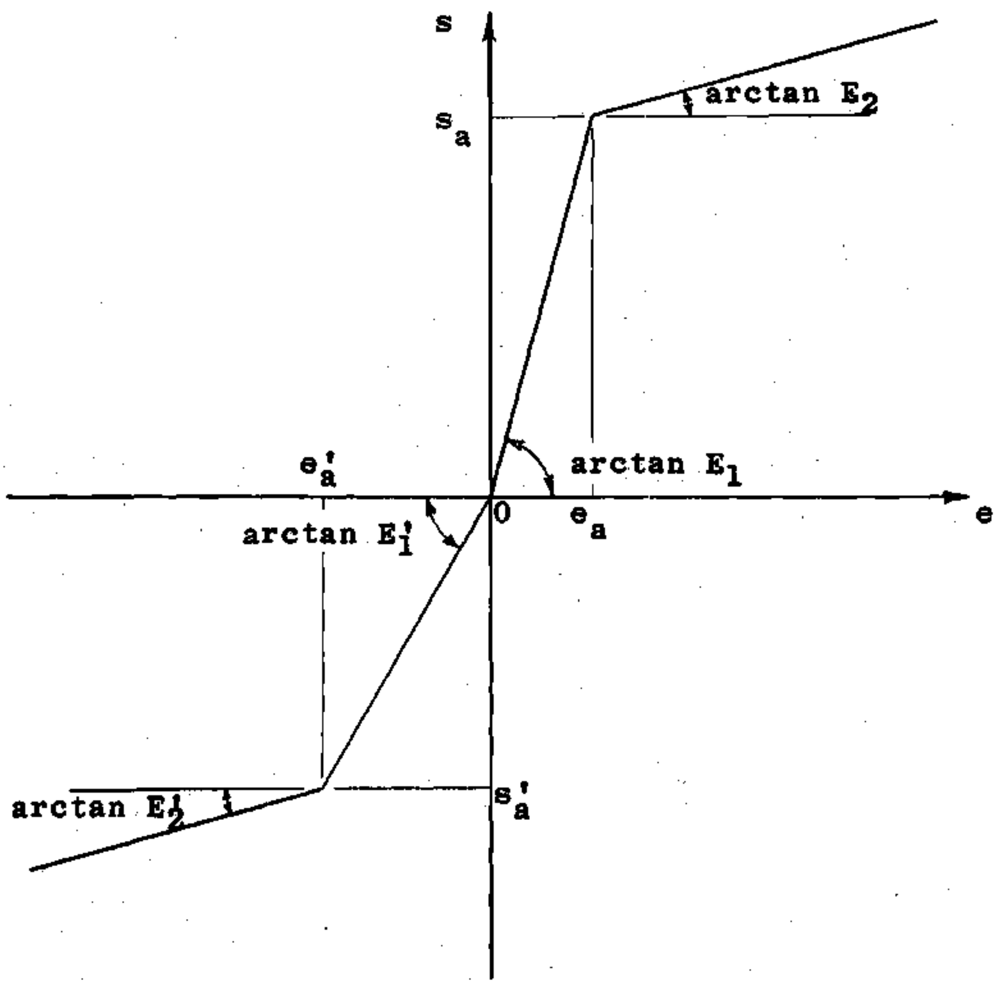

Figure 5. Straight-Line Stress-Strain Curve

Cotterman (2) in an analysis of pure bending of rectangular beams in the plastic range and gave very close agreement with their experimental data.

The curve as indicated in Figure 5 passes through the origin of the stress versus strain plot. The values E_1, E_2, e_a, s_a, E_1', E_2', e_a', and s_a' are constants for materials in any given condition.

The equations defining the stress-strain curve of Figure 5 are

$$\begin{aligned} s &= E_1 e, \quad 0 \leqslant e \leqslant e_a \\ s &= E_1' e, \quad e_a' \leqslant e \leqslant 0 \\ s &= s_a + E_2(e - e_a), \quad e \geqslant e_a \\ s &= s_a' + E_2'(e - e_a'), \quad e \leqslant e_a' \end{aligned} \tag{44}$$

Defining, by Equation (3),

$$y_a = re_a \quad \text{and} \quad y_a' = re_a' \tag{45}$$

Equations (44) become

$$\begin{aligned} s_1 &= E_1 y/r, \quad 0 \leqslant y \leqslant y_a \\ s_1 &= E_1' y/r, \quad y_a' \leqslant y \leqslant 0 \\ s_1 &= s_a + \frac{E_2}{r}(y - y_a), \quad y \geqslant y_a \\ s_1 &= s_a' + \frac{E_2'}{r}(y - y_a'), \quad y \leqslant y_a' \end{aligned} \tag{46}$$

It is now desired to substitute Equations (46) into Equation (42). Because of the nature of Equations (46), it will be necessary to perform the integration indicated in Equation (42) over each region of the stress-strain curve separately. Hence, four separate cases occur.

Case 1: $t_2 \leq y_a$ and $-t_1 \geq y_a'$

Case 2: $t_2 \leq y_a$ and $-t_1 \leq y_a'$

Case 3: $t_2 \geq y_a$ and $-t_1 \geq y_a'$

Case 4: $t_2 \geq y_a$ and $-t_1 \leq y_a'$

If s_a is regarded as the stress at which deformation becomes plastic in tension and s_a' as the stress at which deformation becomes plastic in compression, then Case 1 represents purely elastic bending. Case 2 and Case 3 represent bending in which one side of the member remains completely elastic and the other side undergoes plastic deformation in compression and tension respectively. Case 4 represents bending in which the extreme fibers on both sides of the member undergo plastic deformation. Since Case 1 represents the trivial case of complete springback, it will not be considered further. Case 4 represents the most common state in forming operations and thus has the most practical significance with respect to springback considerations. The latter case will therefore be considered here.

Substituting Equations (46) into Equation (42), subject to the restrictions of Case 4, one obtains

$$M = w\int_{-t_1}^{t_2} s_1 y\,dy = w\left[\int_{-t_1}^{y_a'} \left[s_a' + \frac{E_2'}{r}(y - y_a')\right] y\,dy\right. \qquad (47)$$

$$+ \int_{y_a'}^{0} \frac{E_1'}{r}\, y^2 dy + \int_{0}^{y_a} \frac{E_1}{r}\, y^2 dy$$

$$\left. + \int_{y_a}^{t_2} \left[s_a + \frac{E_2}{r}(y - y_a)\right] y\,dy\right]$$

From Figure 5, it is seen that

$$E_1 e_a = s_a \quad \text{and} \quad E_1' e_a' = s_a'$$

or, substituting Equations (45),

$$E_1 y_a / r = s_a \quad \text{and} \quad E_1' y_a' / r = s_a' \qquad (48)$$

Integrating Equation (47), substituting Equations (48), and simplifying gives

$$\frac{M}{wr^2} = \frac{1}{6}\left\{\frac{s_a}{r^2}\left[3(t_2^{\,2} - y_a^{\,2}) - 3\frac{s_a'}{s_a}(t_1^{\,2} - y_a'^{\,2})\right.\right. \qquad (49)$$

$$\left. + 2y_a^{\,2} - 2\frac{s_a'}{s_a}y_a'^{\,2}\right] + \frac{E_2}{r^3}\left[(2t_2 + y_a)(t_2 - y_a)^2\right.$$

$$\left.\left. + \frac{E_2'}{E_2}(2t_1 - y_a')(t_1 + y_a')^2\right]\right\}$$

Substituting Equations (46) into Equation (41), still subject to the restrictions of Case 4, one obtains

$$\int_{-t_1}^{t_2} s_1 dy = \int_{-t_1}^{y'_a} \left[s'_a + \frac{E'_2}{r}(y - y'_a) \right] dy + \int_{y'_a}^{0} \frac{E'_1}{r} y dy \tag{50}$$

$$+ \int_{0}^{y_a} \frac{E_1}{r} y dy + \int_{y_a}^{t_2} \left[s_a + \frac{E_2}{r}(y - y_a) \right] dy = 0$$

Integrating Equation (50), substituting Equations (48), and simplifying gives

$$\frac{E_2}{2r}(t_2 - y_a)^2 + s_a(t_2 - y_a) + \left[\frac{1}{2}(s_a y_a - s'_a y'_a) \right. \tag{51}$$

$$\left. + s'_a(y'_a + t_1) - \frac{E_2}{2r}(y'_a + t_1)^2 \right] = 0$$

If E_2 is not zero, the solution of Equation (51) for t_2/r is

$$\frac{t_2}{r} = \frac{y_a}{r} - \frac{s_a}{E_2} + \left\{ \left(\frac{s_a}{E_2} \right)^2 \right. \tag{52}$$

$$\left. - \frac{s_a}{E_2} \left[\frac{y_a}{r} - \frac{s'_a y'_a}{s_a r} + \frac{2s'_a}{r s_a}(t_1 + y'_a) \right] + \frac{E'_2}{E_2} \frac{(t_1 + y'_a)^2}{r^2} \right\}^{\frac{1}{2}}$$

If E_2 is zero, then the solution of Equation (51) for t_2/r is

$$\frac{t_2}{r} = \frac{1}{2}\left[\frac{y_a}{r} + \frac{s_a' y_a'}{s_a r}\right] - \frac{s_a'}{s_a}\left[\frac{y_a'}{r} + \frac{t_1}{r}\right] + \frac{E_2'}{2s_a} \quad \frac{(y_a' + t_1)^2}{r^2} \qquad (53)$$

From Figure 3 it is seen that

$$t/r = t_1/r + t_2/r \qquad (54)$$

The preceding equations contain four variables, namely, M, t_1, t_2, and r. Three independent equations have been derived involving these variables: Equations (49), (52) or (53), and (54). Thus, if one variable is specified, the three applicable equations may be solved uniquely for the remaining three unknowns. However, such a solution is exceedingly complex and tedious. For practical computation purposes, it appears simpler to introduce three parameters, t_1/r, t_2/r, and M/wr^2. Then, for any specified value of t_1/r, Equation (52) or (53), as the case may be, may be solved for t_2/r. Equation (49) may then be solved for M/wr^2, and Equation (54) may be solved for t/r.

Equation (43) may be written as

$$\frac{\phi}{\theta} = 1 - \frac{12}{E}\left(\frac{M}{wr^2}\right)\left(\frac{r}{t}\right)^3 \qquad (55)$$

It is to be noted that E in Equation (55) is the actual modulus of elasticity of the material, and is not necessarily equal to any of the straight-line approximation parameters E_1, E_1', E_2, or E_2'.

Therefore, the springback ratio ϕ/θ may be determined for any particular ratio of r/t by the numerical method outlined above. The most practical method of computation is to plot ϕ/θ versus r/t over a considerable range, by assuming various values of t_1/r to start the computation.

<u>Solution Using Exponential Approximation of Stress-Strain Curve</u>.--Tsun Kuei Wang (6) proposed an analytical expression as an approximation to the stress-strain curve as determined by the simple tension-compression test. His relation applies to those materials whose yield strengths are determined by the 0.20 per cent offset method and has the advantage that the parameters involved are easily determined. Wang's expression for the tension curve, with typographical error corrected, is

$$e = s/E + e_u \exp \left[K(s/s_u - 1)\right] , \tag{56}$$

$$\text{where } K = \frac{\ln (0.002/e_u)}{s_y/s_u - 1}$$

Taking the ultimate compressive properties as equal to the ultimate tensile properties, as is customary for wrought ductile materials, the analogous expression for the compression curve is

$$e = s/E - e_u \exp \left[K'(s/-s_u - 1)\right] , \tag{57}$$

$$\text{where } K' = \frac{\ln (0.002/e_u)}{s_y'/-s_u - 1}$$

By Equation (3), Equations (56) and (57) become

$$y/r = s_1/E + e_u \exp\left[K(s_1/s_u - 1)\right], \tag{58}$$

$$\text{where } K = \frac{\ln\,(0.002/e_u)}{s_y/s_u - 1},\quad y \geq 0$$

and

$$y/r = s_1/E - e_u \exp\left[K'(s_1/-s_u - 1)\right], \tag{59}$$

$$\text{where } K' = \frac{\ln\,(0.002/e_u)}{s_y'/-s_u - 1},\quad y \leq 0$$

While Equations (58) and (59) do not pass through the origin of the stress versus strain plot, and therefore form a discontinuous curve, their deviation from the origin is negligible for all practical purposes. Therefore, it will be assumed that for $s_1 = 0$, $y = 0$ for both equations in the following derivation.

Equation (42) may be written as

$$M = w\int_{-t_1}^{t_2} s_1 y\,dy \tag{60}$$

$$= \frac{w}{2}\left[s_1 y^2 \Bigg|_{s_m,-t_1}^{s_M,t_2} - \int_{s_m}^{0} y^2 ds_1 - \int_{0}^{s_M} y^2 ds_1 \right]$$

where

$$s_m = f(-t_1/r) \quad \text{and} \quad s_M = f(t_2/r) \tag{61}$$

Substituting Equations (58) and (59), integrating, and simplifying yields

$$\frac{M}{wr^2} = \frac{1}{3E^2}\,(s_M{}^3 - s_m{}^3) \tag{62}$$

$$+ \frac{e_u}{EK^2}(s_M{}^2K^2 - s_MKs_u + s_u{}^2)\ \exp\left[K(\frac{s_M}{s_u} - 1)\right]$$

$$+ \frac{e_u}{EK'^2}(s_m{}^2K'^2 + s_mK's_u + s_u{}^2)\ \exp\left[K'(\frac{s_m}{-s_u} - 1)\right]$$

$$+ \frac{e_u{}^2}{4K}(2s_MK - s_u)\ \exp\ \left[2K(\frac{s_M}{s_u} - 1)\right]$$

$$- \frac{e_u{}^2}{4K'}(2s_mK' + s_u)\ \exp\left[2K'(\frac{s_m}{-s_u} - 1)\right]$$

Equation (41) may be written as

$$\int_{-t_1}^{t_2} s_1 dy = s_1 y \Big|_{s_m,-t_1}^{s_M,t_2} - \int_{s_m}^{0} y ds_1 - \int_0^{s_M} y ds_1 \tag{63}$$

$$= 0$$

Substituting Equations (58) and (59), integrating, and simplifying yields

$$r\left\{\frac{1}{2E}\left(s_M{}^2 - s_m{}^2\right) + \frac{e_u}{K}(s_M K - s_u)\exp\left[K\left(\frac{s_M}{s_u} - 1\right)\right]\right. \quad (64)$$

$$\left. + \frac{e_u}{K'}(s_m K' + s_u)\exp\left[K'\left(\frac{s_m}{-s_u} - 1\right)\right]\right\} = 0$$

Since r cannot be zero, Equation (64) may be written

$$s_M{}^2/2E + e_u\ (s_M - s_u/K)\ \exp\left[K(s_M/s_u\ - 1)\right] \quad (65)$$

$$= s_m{}^2/2E - e_u(s_m + s_u/K')\ \exp\left[K'(s_m/-s_u\ - 1)\right]$$

From Equations (58), (59), and (61), it is seen that

$$t_2/r = s_M/E + e_u\ \exp\left[K(s_M/s_u - 1)\right] \quad (66)$$

and

$$t_1/r = -s_m/E + e_u\ \exp\left[K'(s_m/-s_u - 1)\right] \quad (67)$$

The preceding equations furnish all the information necessary to compute the springback ratio for any particular case. However, the form of the equations does not permit the springback ratio to be computed explicitly. The simplest method of attack, for practical computation purposes, appears to be as follows:

(1) Assume various values of the maximum and minimum fiber stresses in bending, s_M and s_m.

(2) Plot curves of the left-hand side of Equation (65) versus s_M and the right-hand side of Equation (65) versus s_m on the same axes.

(3) Using the curves just constructed, plot a curve of s_M versus s_m for which Equation (65) is satisfied.

(4) Compute M/wr^2 from Equation (62), using various values of s_M and the corresponding values of s_m found from the curve of step (3).

(5) By Equations (54), (66), and (67), compute t/r for the paired values of s_M and s_m used above.

(6) Substitute into Equation (55) to compute the springback ratio, and plot the curve of the springback ratio ϕ/θ versus r/t.

Change of Variable.--Both analyses developed above give the springback ratio as a function of the thickness and the bent radius of curvature of the neutral surface in bending. Such results are of theoretical value, but, since the location of the neutral surface in bending is not generally known, it would be preferable to determine springback in terms of a part design parameter which can be easily determined.

Letting R_d be the bent radius of curvature of the innermost fiber of the bent number, it is seen from Figure 1 that

$$R_d + t_1 = r \tag{68}$$

Hence,

$$\frac{R_d}{t} = \frac{r}{t}\left(1 - \frac{t_1}{r}\right) \tag{69}$$

Therefore, instead of plotting the springback ratio against r/t, it would be of more practical value to compute R_d/t in one additional computation and to plot the springback ratio against R_d/t.

CHAPTER III

DISCUSSION OF RESULTS

General.--A general expression, Equation (38), which should apply to most engineering materials, has been derived, relating springback to the bent radius of curvature, the cross-section, and the stress-strain diagram of the material of the member.

The springback ratio could be evaluated without further approximation by using the actual stress-strain curve of a material to solve the equations of static equilibrium. However, in order to eliminate graphical integrations, this was not done in the present analysis. Instead, two types of analytical approximation of the stress-strain curve were chosen, and the general equation of springback was evaluated for these approximations.

In order to evaluate the springback of a material it is necessary to know the typical stress-strain curve, in both tension and compression, of the material for the conditions under which it is to be formed. Requests for such information on materials for which springback data are available were made by the author to the following:

Aluminum Company of America

Kaiser Aluminum and Chemical Sales, Inc.

Reynolds Metals Company

Lockheed Aircraft Company, Georgia Division

Wright Air Development Center

None of the above organizations were able to furnish any applicable data. The curves used in the calculations involved in this investigation were those given by Sturm and Fletcher (16). Although not giving complete data, these curves were chosen as being the most extensive and typical data found in the literature.

Comparison of Results with Test Data.--For the three materials investigated, the calculated springback ratio differs from reported test data by less than 10 per cent. In general, the calculated results agree with the test data within 5 per cent in the region of most severe bending, or smaller values of the R_d/t ratio.

It was found that both approximations to the stress-strain curve gave substantially the same results over their applicable range and that the exponential approximation yielded reasonable results over the range of very slight bending.

Although no effort was made to analyze the bending of cladded materials, the calculated results for bare materials agree reasonably well with available test data for 2024-T Alclad and 2024-O Alclad sheets.

A greater difference exists between the calculated results and the test data in the case of 2024-T. It would appear that this variation is attributable to the use of a non-typical stress-strain curve in the calculations. Although it is apparent from data in the Alcoa Aluminum Handbook (29) that the curve used is not typical, this curve was the most accurate and complete one available to the author.

Comparison of Results with Other Theories.--The results of the two approximations presented here were compared with the theories developed by Schroeder (17), Schroeder (22), and Gardiner (23). Schroeder (17) published an equation to determine the springback of 2024-T Alclad. His equation requires double graphical integration of the stress-strain curve, a tedious process, for a solution and applies to only one material. Although this equation was presented for 2024-T Alclad sheet only, it is seen from Figure 16 that there is only slight difference between the results presented here and Schroeder's (17) values over the range which his calculation covers.

The relation derived by Schroeder (22) gives results which agree quite well with those presented here in the region of severe bending. However, his results differ more greatly from those of the author and from the test data as bending becomes less severe. In the region of slight bending, Schroeder's (22) calculation of the springback ratio greatly underestimates the values found in tests of springback.

The springback function derived by Gardiner (23) slightly overestimates the springback ratio in the region of severe bending and is farther in error than those derived by the writer. His results agree fairly well with those of the author and with existing test data in the mid-range of bending, but differ from them more with increasing bend radius. In the region of slight bending Gardiner's results also underestimate the springback ratio, falling between the results of Schroeder (22) and those of the author.

It is to be noted that the first portions of the stress-strain curve approximations used in the various theories discussed are quite different. This fact accounts for the large variation of predicted springback values for large bend radii among the various theories.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Under the assumptions of the classic beam theory, a relatively simple expression (Eq. 38) has been derived for the springback ratio of an initially straight member subjected to pure bending. It has been demonstrated that springback does occur about an axis through the centroid of the cross-section, as has been assumed in several previous analyses. It is to be remembered that these results were obtained by considering only axial stresses and by neglecting any deformation of the cross-section in bending. Thus, it is not expected that the results obtained would apply as well to open sections, such as channels, or to tubes as they do to a rectangular section. However, no calculations were made for such cross-sections, since no test data exist in the literature for such cases.

The general expression for springback was applied to the case of a rectangular cross-section (Eq. 43). Two separate analytical approximations of the stress-strain curve were then chosen, and each was applied to the springback relation for a rectangular cross-section. Calculations for three materials for which springback data are available in the literature showed good agreement between the theoretical results and test data, even for cladded materials. Test data are available for 2024-T sheet over a wider range of severity of bending than for other materials. However, because of the use of a non-typical stress-strain curve for 2024-T, agreement between the theoretical results obtained and test data is not as good as for the other materials considered.

The theoretical results derived here agree with available test data more closely than do those derived by Schroeder (22) and Gardiner (23) throughout the range of bending covered by the data. The equation for springback published by Schroeder (17) for 2024-T Alclad sheet is only slightly more accurate than the theories presented here for the values which he calculated.

It is recommended that the straight-line approximation be used for cases of severe bending, since this method allows an explicit solution. For the region of slight bending, the exponential approximation is recommended, since the straight-line method is not as accurate in this region, and, as developed here, is not valid for cases of very slight bending. Where the accuracy of Gardiner's method is sufficient, his equation is to be preferred because of the smaller amount of computation involved.

The present analysis should not be regarded as a complete solution to the problem of springback. Much further study is needed before such a solution is possible.

Of greatest need is basic knowledge of the plastic behavior of a material subjected to biaxial and triaxial stresses. The manner in which a material behaves upon unloading from a stressed state also needs study.

With particular regard to the problems of metal forming and springback, further work needs to be done to discover the effects of method of loading, width of the formed member, and speed and temperature of forming. This analysis was made under the assumption of loading to produce pure bending. The effect of the width of the member was not considered, since cross-sectional deformations during bending were neglected. The speed and temperature of forming determine the shape of the stress-strain curve of the material and thus relate directly to the position of the neutral axis in bending and to the moment required for bending. The manner in which a material deforms and the quantitive and qualitative effects of the variables involved is a very basic problem which has not yet been satisfactorily explained. Since this behavior determines the stress-strain diagram, no metal-forming problem can be analyzed completely until a deformation theory is complete.

A P P E N D I X

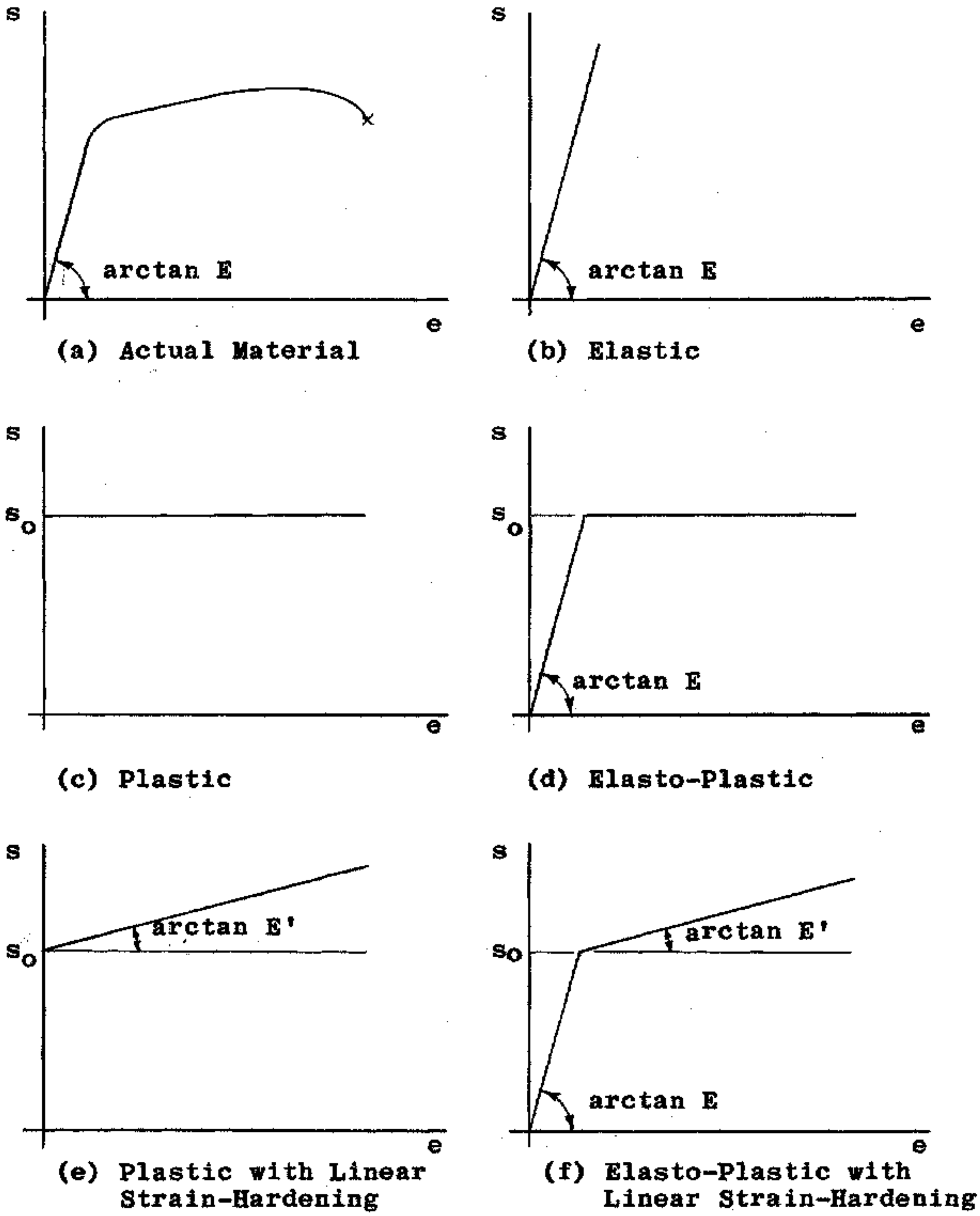

Figure 6. Types of Idealized Stress-Strain Relations

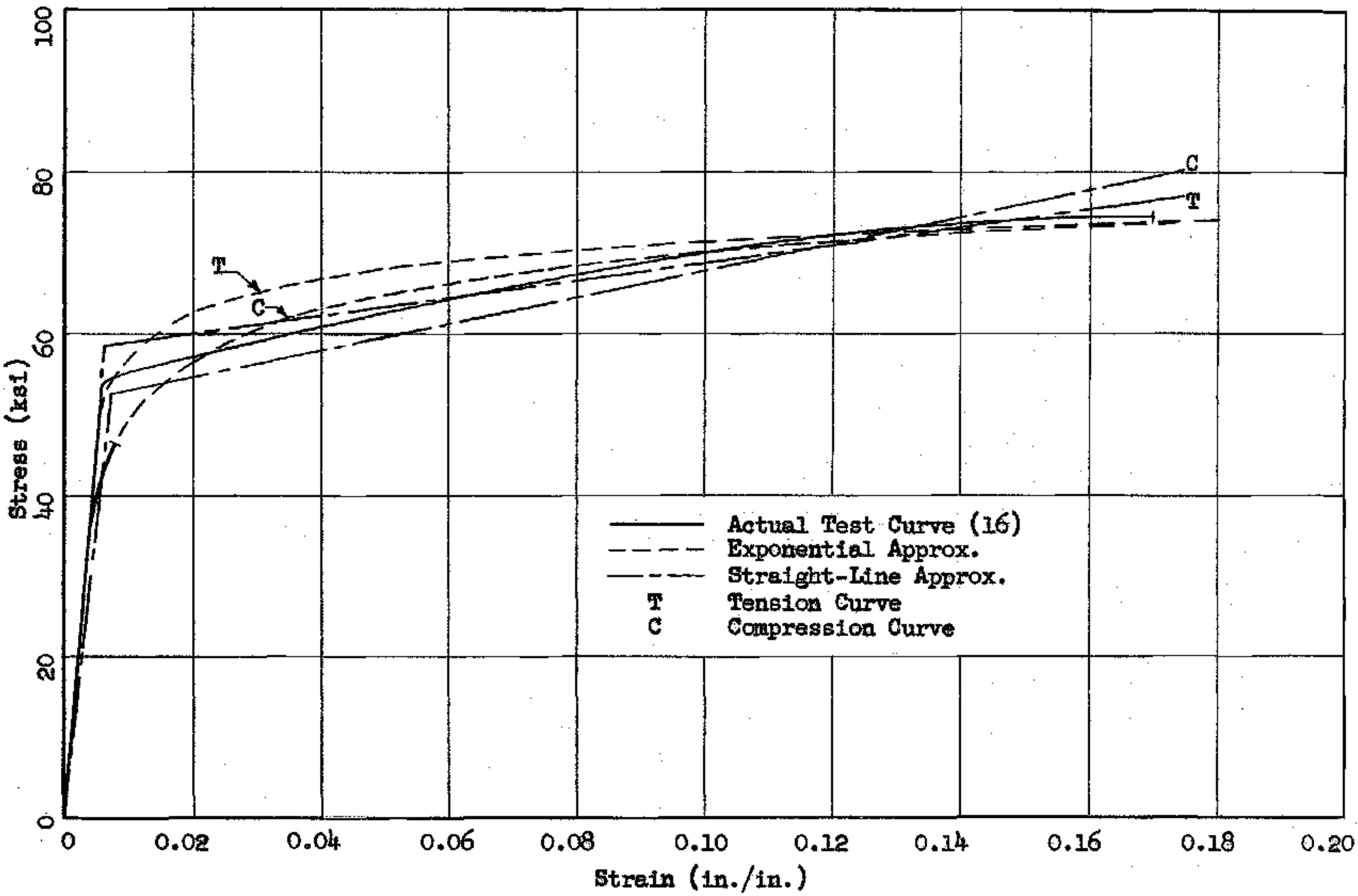

Figure 7. Stress-Strain Curves for 2024-T Aluminum Alloy

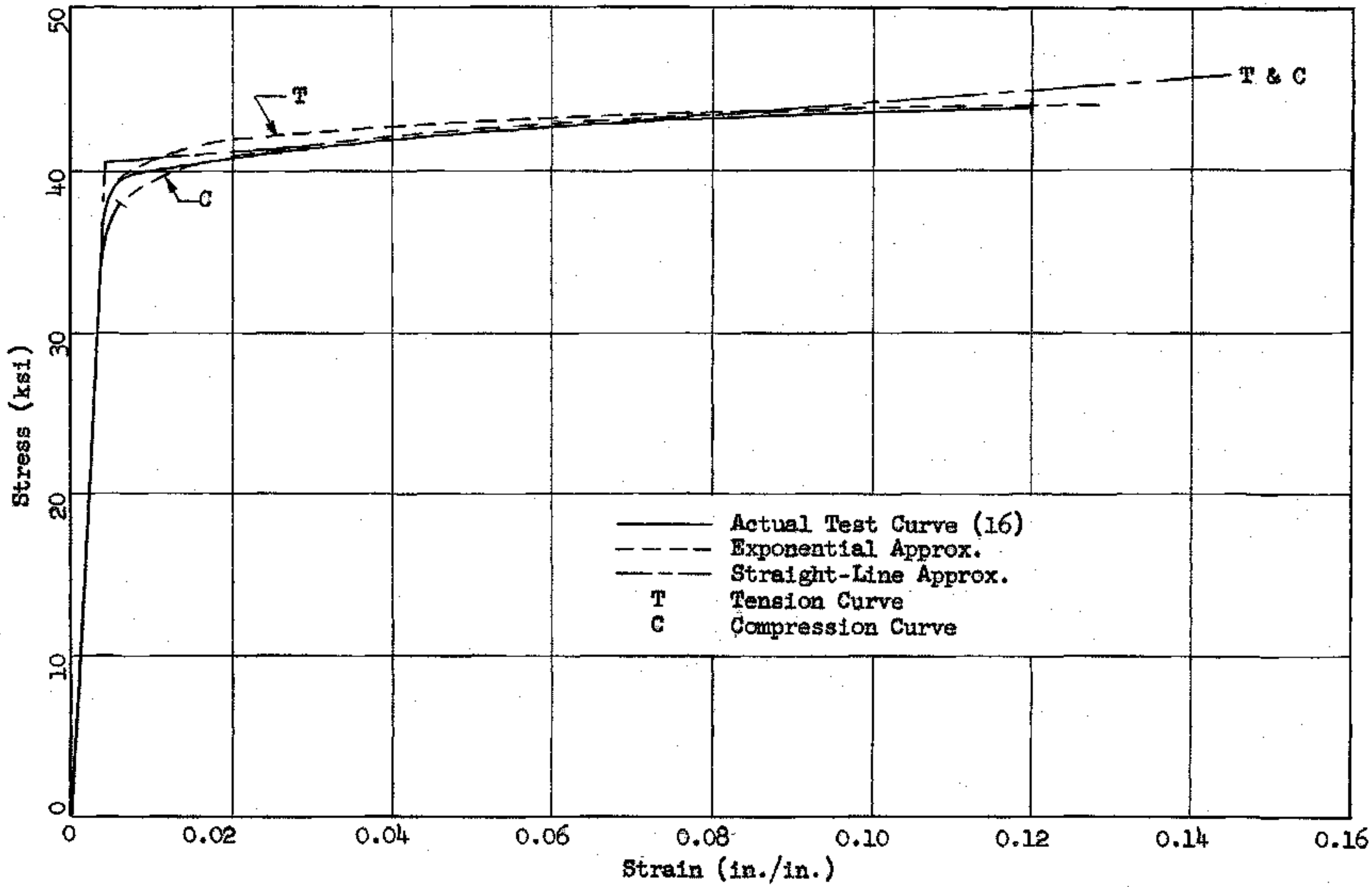

Figure 8. Stress-Strain Curves for 6061-T6 Aluminum Alloy

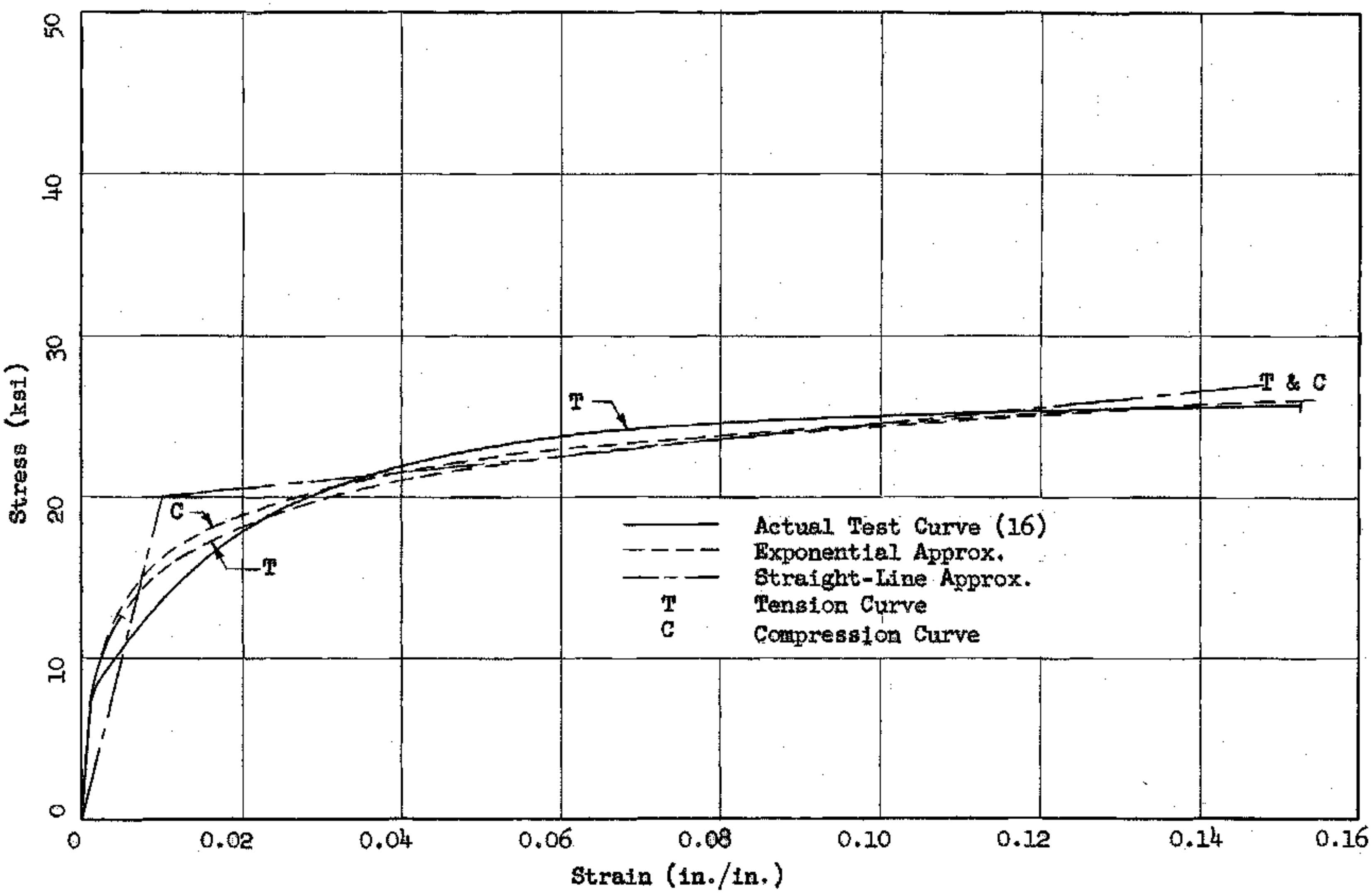

Figure 9. Stress-Strain Curves for 2024-0 Aluminum Alloy

Table 1. Material Properties

Material	s_y* (ksi)	s'_y* (ksi)	s_u* (ksi)	E** (ksi x 1000)	e_u** (in./in.)
2024-T	54.7	-42.2	74.5	10.6	0.18
2024-O	9.0	-10.9	25.8	10.6	0.20
6061-T6	39.5	-37.7	44.0	10.0	0.12

*From stress-strain curves of Sturm and Fletcher (16).
**From table of typical mechanical properties, Alcoa Aluminum Handbook (29).

Table 2. Parameters--Straight-Line Approximation

Material	s_a (ksi)	s'_a (ksi)	E_1 (ksi x 1000)	E_2 (ksi)	E'_1 (ksi x 1000)	E'_2 (ksi)
2024-T	58.5	-52.5	9.0	110.0	7.0	165.0
2024-O	20.0	-20.0	2.0	50.0	2.0	50.0
6061-T6	40.5	-40.5	9.0	37.5	9.0	37.5

Table 3. Calculated Values--Straight-Line Approximation--2024-T

t_1/r (in./in.)	t/r (in./in.)	M/wr^2 (psi)	ϕ/θ	R_d/t (in./in.)
0.0010000	0.0018112	-0.84277	161.58	551.57
0.0012589	0.0023002	-0.81538	76.842	434.19
0.0015849	0.0029161	-0.77160	36.227	342.38
0.0019953	0.0036915	-0.70170	16.791	270.35
0.0025119	0.0046681	-0.59020	7.5683	213.68
0.0031623	0.0058980	-0.41244	3.2757	169.01
0.0039811	0.0074472	-0.12914	1.3540	133.74
0.0050119	0.0093988	0.32241	0.56039	105.86
0.0063096	0.011858	1.0424	0.29224	83.801
0.0079433	0.014956	2.1910	0.25863	66.329
0.010000	0.018862	4.0255	0.32096	52.485
0.012589	0.023788	6.9593	0.41469	41.509
0.015849	0.030000	11.660	0.51114	32.805
0.019953	0.037840	19.208	0.59868	25.900
0.025119	0.047740	31.365	0.67365	20.421
0.031623	0.060247	51.016	0.73590	16.073
0.039811	0.076063	82.924	0.78668	12.624
0.050119	0.096079	135.01	0.82767	9.8864
0.063096	0.12144	220.59	0.86055	7.7152
0.079433	0.15360	362.27	0.88682	5.9934
0.10000	0.19443	598.91	0.90775	4.6290
0.12589	0.24632	998.15	0.92439	3.5486
0.15849	0.31234	1679.4	0.93761	2.6942
0.19953	0.39640	2856.0	0.94809	2.0194
0.25119	0.50346	4915.4	0.95639	1.4873
0.31623	0.63980	8568.7	0.96296	1.0687
0.39811	0.81336	15139.	0.96815	0.74001
0.50119	1.0341	27114.	0.97224	0.48238
0.63096	1.3144	49220.	0.97546	0.28077
0.79433	1.6700	90507.	0.97800	0.12316
1.00000	2.1202	168440.	0.97999	0.00000

Table 4. Calculated Values--Straight-Line Approximation--6061-T6

t_1/r (in./in.)	t/r (in./in.)	M/wr^2 (psi)	$\emptyset/\Theta$	R_d/t (in./in.)
0.0010000	0.0020000	-0.23188	35.782	499.50
0.0012589	0.0025178	-0.20827	16.658	396.67
0.0015849	0.0031698	-0.17083	7.4365	314.98
0.0019953	0.0039906	-0.11147	3.1050	250.09
0.0025119	0.0050238	-0.017365	1.1644	198.55
0.0031623	0.0063246	0.13187	0.37449	157.61
0.0039811	0.0079622	0.36855	0.12383	125.09
0.0050119	0.010024	0.74399	0.11355	99.263
0.0063096	0.012619	1.3397	0.20001	78.744
0.0079433	0.015886	2.2850	0.31612	62.446
0.010000	0.020000	3.7859	0.43212	49.500
0.012589	0.025178	6.1697	0.53617	39.216
0.015849	0.031698	9.9580	0.62480	31.048
0.019953	0.039906	15.982	0.69819	24.559
0.025119	0.050238	25.571	0.75798	19.405
0.031623	0.063246	40.850	0.80623	15.311
0.039811	0.079622	65.226	0.84494	12.059
0.050119	0.10024	104.18	0.87587	9.4763
0.063096	0.12619	166.57	0.90053	7.4245
0.079433	0.15886	266.73	0.92017	5.7946
0.10000	0.20000	428.04	0.93579	4.5000
0.12589	0.25178	688.82	0.94822	3.4716
0.15849	0.31698	1112.3	0.95809	2.6548
0.19953	0.39906	1803.9	0.96593	2.0059
0.25119	0.50238	2940.7	0.97217	1.4905
0.31623	0.63246	4823.4	0.97712	1.0811
0.39811	0.79622	7969.2	0.98105	0.75594
0.50119	1.0024	13278.	0.98418	0.49763
0.63096	1.2619	22336.	0.98666	0.29245
0.79433	1.5886	37977.	0.98863	0.12946
1.00000	2.0000	65331.	0.99020	0.00000

Table 5. Calculated Values--Straight-Line Approximation--2024-0

t_1/r (in./in.)	t/r (in./in.)	M/wr^2 (psi)	Ø/θ	R_d/t (in./in.)
0.0010000	0.0020000	-0.63047	90.217	499.50
0.0012589	0.0025178	-0.61903	44.903	396.66
0.0015849	0.0031698	-0.60088	22.359	314.98
0.0019953	0.0039906	-0.57210	11.192	250.09
0.0025119	0.0050238	-0.52644	5.7004	198.55
0.0031623	0.0063246	-0.45395	3.0314	157.61
0.0039811	0.0079622	-0.33884	1.7599	125.09
0.0050119	0.010024	-0.15599	1.1753	99.263
0.0063096	0.012619	0.13468	0.92412	78.745
0.0079433	0.015886	0.59707	0.83142	62.446
0.010000	0.020000	1.3333	0.81132	49.500
0.012589	0.025178	2.5070	0.82219	39.216
0.015849	0.031698	4.3809	0.84428	31.048
0.019953	0.039906	7.3779	0.86856	24.559
0.025119	0.050238	12.182	0.89123	19.405
0.031623	0.063246	19.904	0.91093	15.311
0.039811	0.079622	32.359	0.92743	12.059
0.050119	0.10024	52.528	0.94096	9.4763
0.063096	0.12619	85.354	0.95192	7.4245
0.079433	0.15886	139.09	0.96073	5.7946
0.10000	0.20000	227.68	0.96778	4.5000
0.12589	0.25178	374.91	0.97341	3.4716
0.15849	0.31698	621.87	0.97790	2.6548
0.19953	0.39906	1040.4	0.98146	2.0059
0.25119	0.50238	1758.0	0.98430	1.4905
0.31623	0.63246	3003.4	0.98656	1.0811
0.39811	0.79622	5193.1	0.98835	0.75594
0.50119	1.0024	9093.9	0.98978	0.49763
0.63096	1.2619	16135.	0.99091	0.29245
0.79433	1.5886	29009.	0.99181	0.12946
1.00000	2.0000	52833.	0.99252	0.00000

Table 6. Evaluation of Equation (65) for 2024-T

s_M & $-s_m$ (ksi)	L.H. Side of Eq.(65) (psi)	R.H. Side of Eq.(65) (psi)	s_M & $-s_m$ (ksi)	L.H. Side of Eq.(65) (psi)	R.H. Side of Eq.(65) (psi)
1	0.05714	0.00743	43	92.622	167.31
2	0.18865	0.15040	44	98.281	185.95
3	0.42451	0.38902	45	104.48	207.25
4	0.75471	0.72367	46	111.33	231.64
5	1.1793	1.1548	47	119.00	259.64
6	1.6982	1.6829	48	127.70	291.84
7	2.3114	2.3087	49	137.68	328.96
8	3.0190	3.0329	50	149.26	371.80
9	3.8210	3.8565	51	162.89	421.33
10	4.7174	4.7806	52	179.09	478.66
11	5.7082	5.8065	53	198.55	545.11
12	6.7934	6.9360	54	222.16	622.18
13	7.9730	8.1710	55	251.03	711.67
14	9.2471	9.5137	56	286.60	815.63
15	10.616	10.967	57	330.68	936.49
16	12.079	12.534	58	385.61	1077.1
17	13.637	14.219	59	454.35	1240.6
18	15.290	16.026	60	540.67	1431.0
19	17.037	17.962	61	649.38	1652.7
20	18.880	20.032	62	786.60	1910.9
21	20.818	22.244	63	960.13	2211.6
22	22.851	24.608	64	1179.9	2561.9
23	24.980	27.134	65	1458.5	2970.0
24	27.206	29.835	66	1812.1	3445.4
25	29.529	32.727	67	2261.0	3999.3
26	31.950	35.827	68	2831.4	4644.6
27	34.470	39.157	69	3556.2	5396.5
28	37.090	42.741	70	4477.6	6272.3
29	39.813	46.609	71	5649.0	7292.5
30	42.640	50.794	72	7138.5	8480.9
31	45.574	55.339	73	9032.4	9865.0
32	48.619	60.290	74	11441.	11477.
33	51.781	65.703	75	14502.	13354.
34	55.064	71.644	76	18396.	15539.
35	58.479	78.191	77	23345.	18084.
36	62.034	85.434	78	29636.	21046.
37	65.743	93.479	79	37633.	24493.
38	69.624	102.45	80	47796.	28505.
39	73.697	112.50	81	60710.	33174.
40	77.993	123.79	82	77120.	38606.
41	82.546	136.52	83	97967.	44926.
42	87.402	150.94	84	124450.	52277.
			85	158080.	60826.

Table 7. Evaluation of Equation (65) for 6061-T6

s_M & $-s_m$ (ksi)	L.H. Side of Eq.(65) (psi)	R.H. Side of Eq.(65) (psi)	s_M & $-s_m$ (ksi)	L.H. Side of Eq.(65) (psi)	R.H. Side of Eq.(65) (psi)
1.0	0.05000	0.05000	26.5	35.113	35.147
2.0	0.20000	0.20000	27.0	36.450	36.499
3.0	0.45000	0.45000	27.5	37.813	37.881
4.0	0.80000	0.80000	28.0	39.202	39.297
5.0	1.2500	1.2500	28.5	40.615	40.749
6.0	1.8000	1.8000	29.0	42.054	42.242
7.0	2.4500	2.4500	29.5	43.519	43.784
8.0	3.2000	3.2000	30.0	45.010	45.382
9.0	4.0500	4.0500	30.5	46.529	47.050
10.0	5.0000	5.0000	31.0	48.076	48.807
11.0	6.0500	6.0500	31.5	49.654	50.678
12.0	7.2000	7.2000	32.0	51.267	52.700
12.5	7.8125	7.8125	32.5	52.920	54.922
13.0	8.4500	8.4500	33.0	54.622	57.416
13.5	9.1125	9.1125	33.5	56.388	60.283
14.0	9.8000	9.8000	34.0	58.242	63.662
14.5	10.512	10.512	34.5	60.219	67.751
15.0	11.250	11.250	35.0	62.380	72.825
15.5	12.012	12.012	35.5	64.820	79.271
16.0	12.800	12.800	36.0	67.690	87.632
16.5	13.612	13.612	36.5	71.232	98.670
17.0	14.450	14.450	37.0	75.834	113.45
17.5	15.312	15.312	37.5	82.112	133.47
18.0	16.200	16.200	38.0	91.052	160.82
18.5	17.112	17.113	38.5	104.23	198.44
19.0	18.050	18.050	39.0	124.15	250.44
19.5	19.012	19.013	39.5	154.82	322.59
20.0	20.000	20.000	40.0	202.62	422.94
20.5	21.012	21.013	40.5	277.76	562.80
21.0	22.050	22.051	41.0	396.47	757.98
21.5	23.112	23.114	41.5	584.67	1031.6
22.0	24.200	24.202	42.0	883.68	1411.7
22.5	25.312	25.315	42.5	1359.4	1944.6
23.0	26.450	26.453	43.0	2116.7	2690.1
23.5	27.612	27.617	43.5	3323.0	3733.0
24.0	28.800	28.806	44.0	5244.9	5192.2
24.5	30.012	30.021	44.5	8307.3	7233.8
25.0	31.250	31.262	45.0	13187.	10090.
25.5	32.513	32.530	45.5	20962.	14087.
26.0	33.800	33.824	46.0	33350.	19679.

Table 8. Evaluation of Equation (65) for 2024-0

s_M & $-s_m$ (ksi)	L.H. Side of Eq.(65) (psi)	R.H. Side of Eq.(65) (psi)	s_M & $-s_m$ (ksi)	L.H. Side of Eq.(65) (psi)	R.H. Side of Eq.(65) (psi)
2.4	-0.15703	0.13377	16.4	167.21	126.41
2.8	0.02988	0.27756	16.8	190.46	145.23
3.2	0.25690	0.45007	17.2	216.83	166.84
3.6	0.52851	0.65399	17.6	246.76	191.65
4.0	0.84984	0.89249	18.0	280.70	220.13
4.4	1.2268	1.1693	18.4	319.18	252.82
4.8	1.6663	1.4886	18.8	362.80	290.32
5.2	2.1761	1.8556	19.2	412.22	333.34
5.6	2.7651	2.2761	19.6	468.22	382.69
6.0	3.4438	2.7569	20.0	531.63	439.27
6.4	4.2238	3.3060	20.4	603.43	504.14
6.8	5.1187	3.9326	20.8	684.70	578.49
7.2	6.1440	4.6473	21.2	776.66	663.69
7.6	7.3172	5.4625	21.6	880.70	761.30
8.0	8.6585	6.3927	22.0	998.36	873.10
8.4	10.191	7.4543	22.4	1131.4	1001.1
8.8	11.940	8.6667	22.8	1281.8	1147.7
9.2	13.937	10.052	23.2	1451.8	1315.4
9.6	16.214	11.636	23.6	1643.8	1507.4
10.0	18.812	13.447	24.0	1860.7	1727.0
10.4	21.772	15.521	24.4	2105.7	1978.2
10.8	25.147	17.896	24.8	2382.2	2265.4
11.2	28.991	20.618	25.2	2694.4	2593.8
11.6	33.370	23.739	25.6	3046.8	2969.2
12.0	38.357	27.318	26.0	3444.3	3398.1
12.4	44.034	31.425	26.4	3892.7	3888.2
12.8	50.497	36.139	26.8	4398.5	4448.1
13.2	57.850	41.550	27.2	4968.7	5087.5
13.6	66.216	47.764	27.6	5611.6	5817.6
14.0	75.731	54.901	28.0	6336.3	6651.1
14.4	86.550	63.098	28.4	7152.9	7602.6
14.8	98.848	72.514	28.8	8073.0	8688.3
15.2	112.82	83.330	29.2	9109.5	9927.2
15.6	128.70	95.754	29.6	10277.	11340.
16.0	146.74	110.02	30.0	11592.	12952.

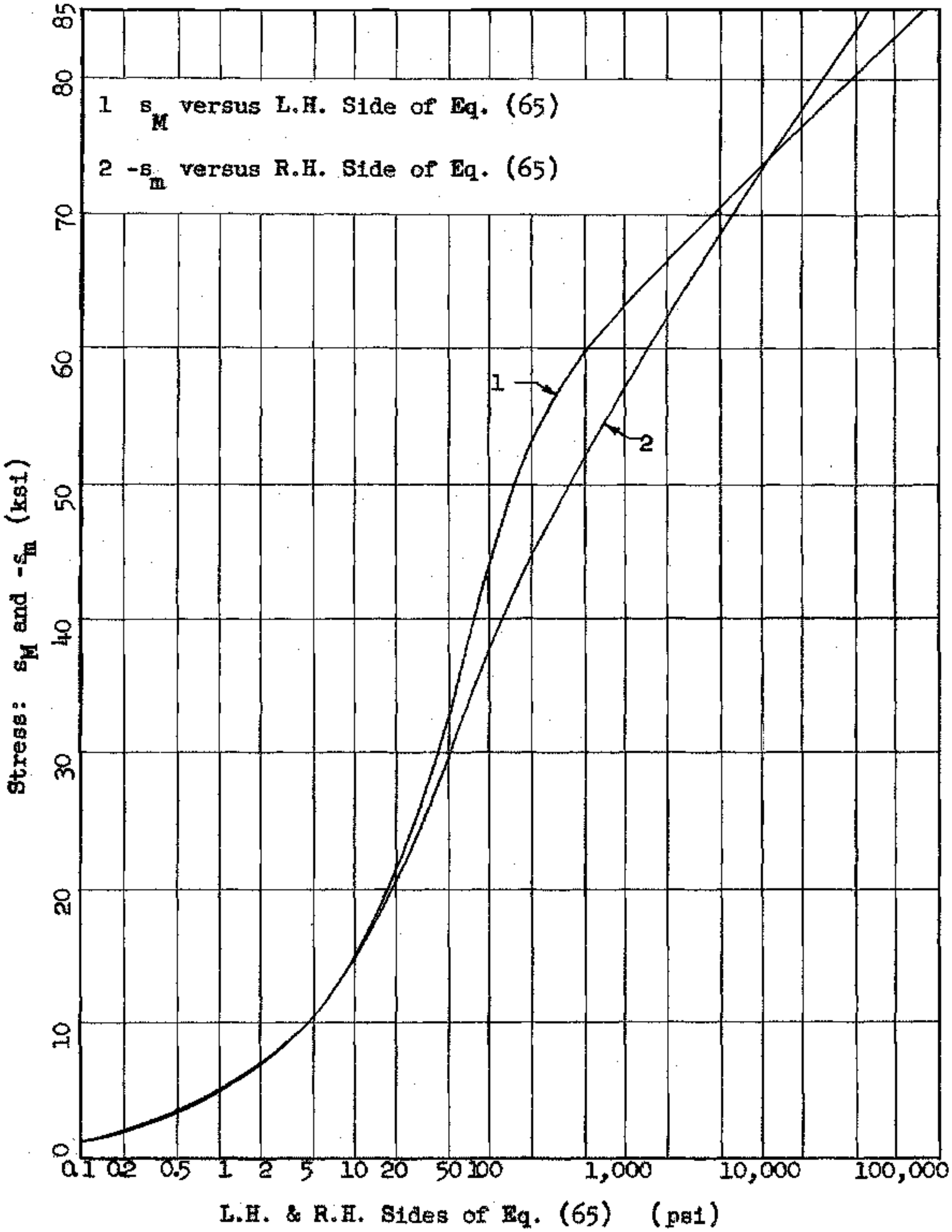

Figure 10. Evaluation of Eq. (65) for 2024-T--
Exponential Approximation

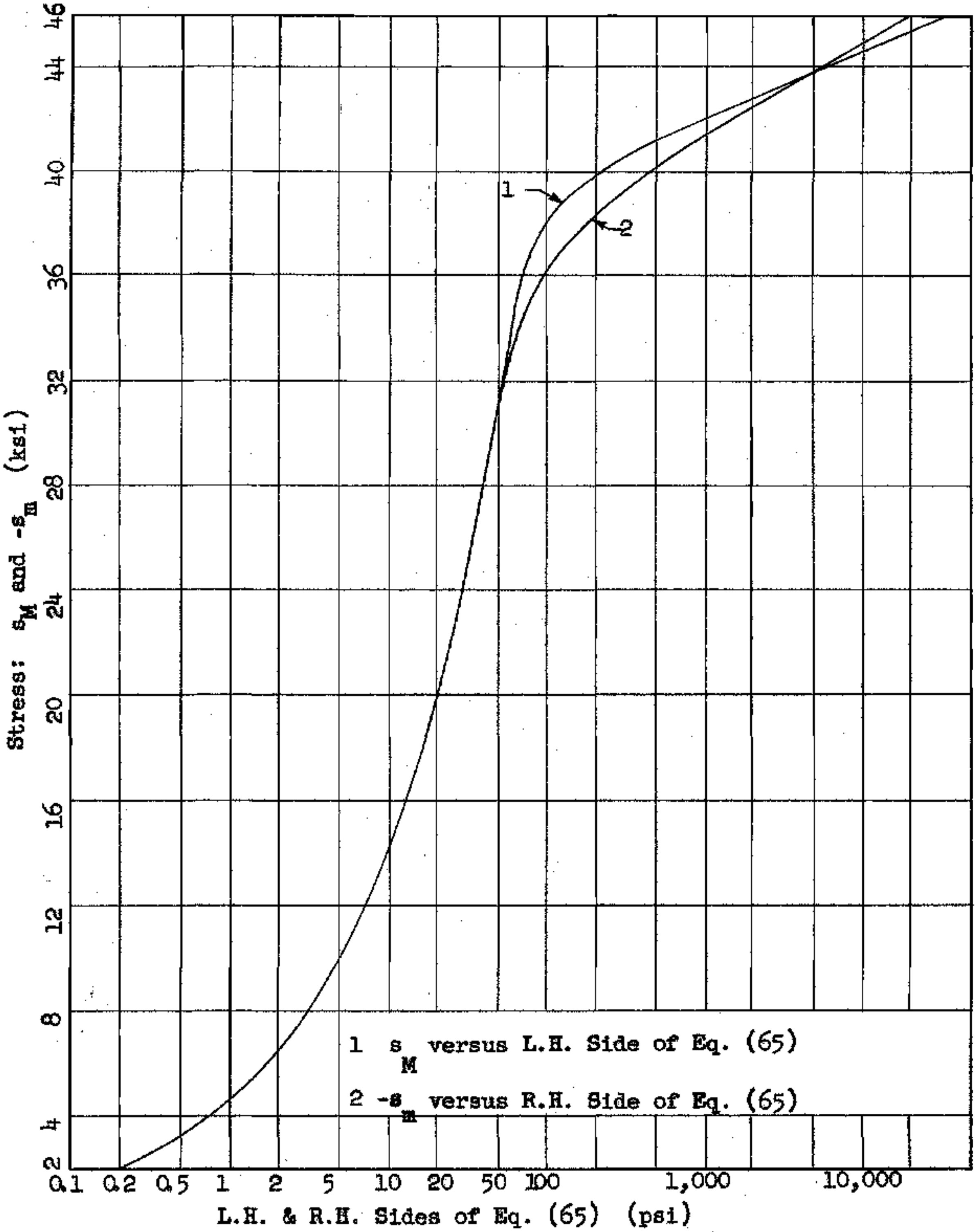

Figure 11. Evaluation of Eq. (65) for 6061-T6--
Exponential Approximation

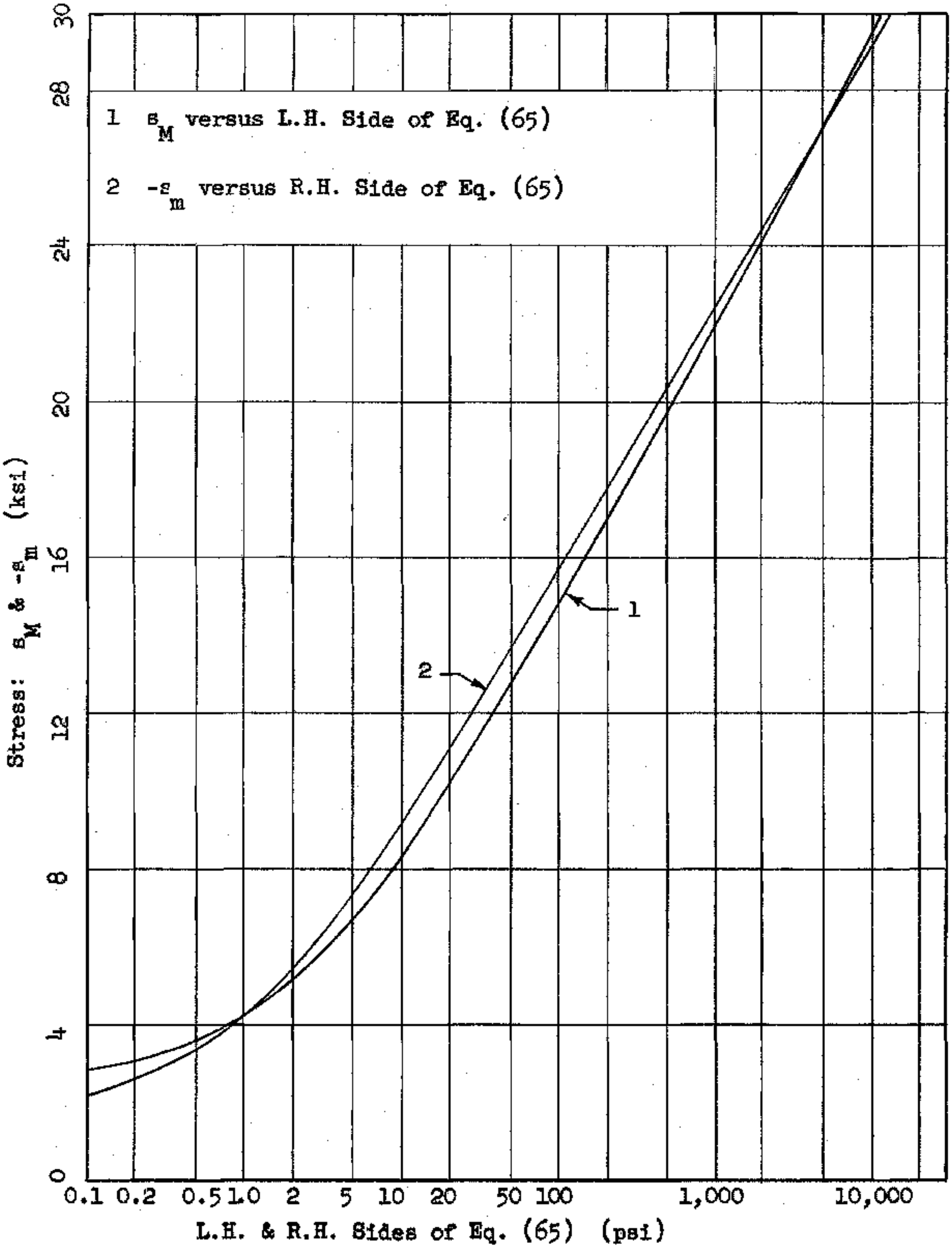

Figure 12. Evaluation of Eq. (65) for 2024-0--
Exponential Approximation

Table 9a. Calculated Values--Exponential Approximation--2024-T

s_M (ksi)	$-s_m$ (ksi)	M/wr^2 (psi)	t_1/r (in./in.)
4	4.073	0.0004266	0.00039411
5	5.046	0.0007956	0.00048734
7	7.004	0.0021078	0.00067560
10	9.932	0.0060414	0.00095930
12	11.874	0.010394	0.0011494
15	14.758	0.020237	0.0014360
20	19.443	0.047915	0.0019182
25	23.887	0.093784	0.0024095
30	27.972	0.16296	0.0029144
35	31.634	0.26214	0.0034433
40	34.970	0.40537	0.0040295
45	38.202	0.63162	0.0047498
47	39.576	0.77013	0.0051212
50	41.884	1.0911	0.0058652
52	43.632	1.4452	0.0065578
55	46.692	2.4531	0.0081444
57	49.040	3.8123	0.0098129
60	52.933	8.5231	0.013914
61	54.304	11.568	0.015921
62	55.721	16.013	0.018412
63	57.168	22.558	0.021486
64	58.629	32.249	0.025257
65	60.124	46.873	0.029965
66	61.617	68.864	0.035723
67	63.141	102.61	0.042941
68	64.660	154.14	0.051801
69	66.200	234.03	0.062882
70	67.741	357.74	0.076590
71	69.288	550.43	0.093619
72	70.849	852.72	0.11492
73	72.410	1326.4	0.14136
74	73.977	2072.0	0.17433
75	75.526	3237.1	0.21478
76	77.105	5091.1	0.26602
77	78.667	8001.9	0.32907
78	80.242	12620.	0.40814
79	81.821	19937.	0.50685
80	83.390	31482.	0.62894
81	84.986	49942.	0.78374

Table 9b. Calculated Values--Exponential Approximation--2024-T

s_M (ksi)	t/r (in./in.)	Ø/Θ	R_d/t (in./in.)
4	0.00077149	-0.051648	1295.7
5	0.00095906	-0.021012	1042.2
7	0.0013360	-0.000639	747.99
10	0.0019028	0.007223	525.04
12	0.0022816	0.009372	437.78
15	0.0028513	0.011726	350.21
20	0.0038058	0.015941	262.25
25	0.0047703	0.021928	209.12
30	0.0057519	0.030580	173.35
35	0.0067679	0.042718	147.25
40	0.0078739	0.059947	126.49
45	0.0092158	0.086440	107.99
47	0.0099028	0.10221	100.46
50	0.011269	0.13698	88.215
52	0.012546	0.17158	79.182
55	0.015474	0.25052	64.097
57	0.018563	0.32534	53.341
60	0.026245	0.46626	37.572
61	0.030048	0.51731	32.750
62	0.034769	0.56871	28.232
63	0.040619	0.61896	24.090
64	0.047850	0.66676	20.371
65	0.056877	0.71160	17.055
66	0.068031	0.75240	14.174
67	0.081998	0.78930	11.672
68	0.099305	0.82181	9.5483
69	0.12096	0.85032	7.7470
70	0.14793	0.87489	6.2423
71	0.18157	0.89590	4.9920
72	0.22370	0.91376	3.9566
73	0.27625	0.92878	3.1082
74	0.34198	0.94135	2.4144
75	0.42352	0.95176	1.8540
76	0.52631	0.96047	1.3946
77	0.65404	0.96762	1.0258
78	0.81427	0.97354	0.72686
79	1.0148	0.97840	0.48595
80	1.2647	0.98238	0.29339
81	1.5799	0.98566	0.13688

Table 10a. Calculated Values--Exponential Approximation--6061-T6

s_M (ksi)	$-s_m$ (ksi)	M/wr^2 (psi)	t_1/r (in./in.)
5.0	5.000	0.0008333	0.00050000
7.5	7.500	0.0028125	0.00075000
10.0	10.000	0.0066667	0.0010000
12.5	12.500	0.013021	0.0012500
15.0	15.000	0.022500	0.0015000
17.5	17.500	0.035729	0.0017500
20.0	20.000	0.053334	0.0020000
22.5	22.500	0.075942	0.0022501
25.0	25.000	0.10420	0.0025005
27.0	26.998	0.13134	0.0027017
28.5	28.454	0.15434	0.0028503
30.0	29.834	0.17957	0.0029954
31.0	30.792	0.19874	0.0031016
32.0	31.646	0.21884	0.0032037
33.0	32.432	0.24068	0.0033084
34.0	33.144	0.26493	0.0034179
35.0	33.810	0.29360	0.0035406
36.0	34.499	0.33244	0.0036997
37.0	35.233	0.39342	0.0039258
38.0	36.150	0.51796	0.0043454
39.0	37.267	0.83186	0.0052362
40.0	38.540	1.8092	0.0073064
40.5	39.189	3.0186	0.0091826
41.0	39.868	5.5244	0.012170
41.5	40.556	10.935	0.016853
42.0	41.230	22.870	0.023955
42.5	41.931	51.044	0.035469
43.0	42.615	117.11	0.053045
43.5	43.303	276.68	0.080618
44.0	44.001	670.21	0.12448
44.5	44.645	1588.9	0.18695
45.0	45.039	3310.6	0.24024
45.418	46.000	8826.2	0.44482

Table 10b. Calculated Values--Exponential Approximation--6061-T6

s_M (ksi)	t/r (in./in.)	Ø/Θ	R_d/t (in./in.)
5.0	0.0010000	0.00000000	999.50
7.5	0.0015000	0.00000000	666.17
10.0	0.0020000	0.00000000	499.50
12.5	0.0025000	0.00000010	399.50
15.0	0.0030000	0.00000010	332.83
17.5	0.0035000	0.00000030	285.21
20.0	0.0040000	0.00000110	249.50
22.5	0.0045001	0.00000440	221.72
25.0	0.0050005	0.00001810	199.48
27.0	0.0054017	0.00005750	184.62
28.5	0.0057004	0.00013580	174.93
30.0	0.0059958	0.00030860	166.28
31.0	0.0062025	0.00056290	160.72
32.0	0.0064059	0.00097520	155.61
33.0	0.0066138	0.0016617	150.70
34.0	0.0068314	0.0027867	145.88
35.0	0.0070740	0.0047116	140.86
36.0	0.0073825	0.0085254	134.95
37.0	0.0078314	0.017105	127.19
38.0	0.0086563	0.041733	115.02
39.0	0.010405	0.11389	95.603
40.0	0.014458	0.28170	68.658
40.5	0.018201	0.39921	54.438
41.0	0.024100	0.52640	40.988
41.5	0.033343	0.64602	29.486
42.0	0.047604	0.74559	20.504
42.5	0.070372	0.82424	13.706
43.0	0.10565	0.88084	8.9627
43.5	0.16111	0.92060	5.7066
44.0	0.24888	0.94783	3.5179
44.5	0.38053	0.96540	2.1366
45.0	0.54282	0.97516	1.3996
45.418	0.88537	0.98474	0.62705

Table 11a. Calculated Values--Exponential Approximation--2024-0

s_M (ksi)	$-s_m$ (ksi)	M/wr^2 (psi)	t_1/r (in./in.)
2.4	1.114	0.00046710	0.00020078
2.8	2.046	0.00062349	0.00031902
4.0	3.928	0.0016074	0.00059023
4.8	4.994	0.0029324	0.00077207
6.0	6.488	0.0067230	0.0010799
6.8	7.431	0.011227	0.0013191
8.0	8.797	0.023308	0.0017552
8.8	9.667	0.037156	0.0021083
10.0	10.934	0.073544	0.0027708
10.8	11.757	0.11515	0.0033270
12.0	12.960	0.22418	0.0043865
12.8	13.753	0.34958	0.0052963
14.0	14.919	0.67984	0.0070502
14.8	15.687	1.0602	0.0085593
16.0	16.824	2.0672	0.011492
16.8	17.581	3.2359	0.014044
18.0	18.697	6.3294	0.018985
18.4	19.068	7.9212	0.021014
18.8	19.438	9.9149	0.023268
19.2	19.809	12.419	0.025785
19.6	20.178	15.553	0.028574
20.0	20.548	19.489	0.031688
20.4	20.917	24.423	0.035148
20.8	21.286	30.614	0.039003
21.2	21.655	38.385	0.043297
22.0	22.391	60.346	0.053383
22.8	23.120	94.736	0.065768
24.0	24.213	186.48	0.090098
24.8	24.942	293.02	0.11126
26.0	26.038	577.71	0.15299
26.8	26.764	907.03	0.18906
27.2	27.126	1136.1	0.21014
27.6	27.487	1422.6	0.23353
28.0	27.849	1781.7	0.25962
28.4	28.211	2231.4	0.28865
28.8	28.573	2794.4	0.32096
29.2	28.936	3500.3	0.35701
29.6	29.299	4384.2	0.39714
30.0	29.662	5490.8	0.44180

Table 11b. Calculated Values--Exponential Approximation--2024-0

s_M (ksi)	t/r (in./in.)	Ø/θ	R_d/t (in./in.)
2.4	0.00073415	-0.33637	1361.8
2.8	0.00092424	0.10596	1081.6
4.0	0.0014354	0.38475	696.24
4.8	0.0018025	0.43313	554.36
6.0	0.0024382	0.47495	409.69
6.8	0.0029388	0.49923	339.83
8.0	0.0038516	0.53819	259.18
8.8	0.0045949	0.56642	217.17
10.0	0.0059863	0.61190	166.58
10.8	0.0071509	0.64350	139.38
12.0	0.0093664	0.69114	106.30
12.8	0.011254	0.72236	88.386
14.0	0.014887	0.76673	66.699
14.8	0.018000	0.79419	55.080
16.0	0.024036	0.83147	41.126
16.8	0.029252	0.85365	33.705
18.0	0.039370	0.88258	24.918
18.4	0.043513	0.89116	22.498
18.8	0.048112	0.89921	20.301
19.2	0.053230	0.90678	18.302
19.6	0.058904	0.91385	16.492
20.0	0.065220	0.92047	14.847
20.4	0.072234	0.92664	13.357
20.8	0.080033	0.93239	12.007
21.2	0.088706	0.93774	10.785
22.0	0.10905	0.94732	8.6806
22.8	0.13408	0.95551	6.9677
24.0	0.18312	0.96562	4.9690
24.8	0.22564	0.97113	3.9387
26.0	0.30913	0.97786	2.7399
26.8	0.38132	0.98148	2.1266
27.2	0.42352	0.98307	1.8650
27.6	0.47037	0.98452	1.6295
28.0	0.52253	0.98586	1.4169
28.4	0.58052	0.98709	1.2254
28.8	0.64499	0.98821	1.0528
29.2	0.71678	0.98924	0.89706
29.6	0.79661	0.99018	0.75679
30.0	0.88538	0.99104	0.63046

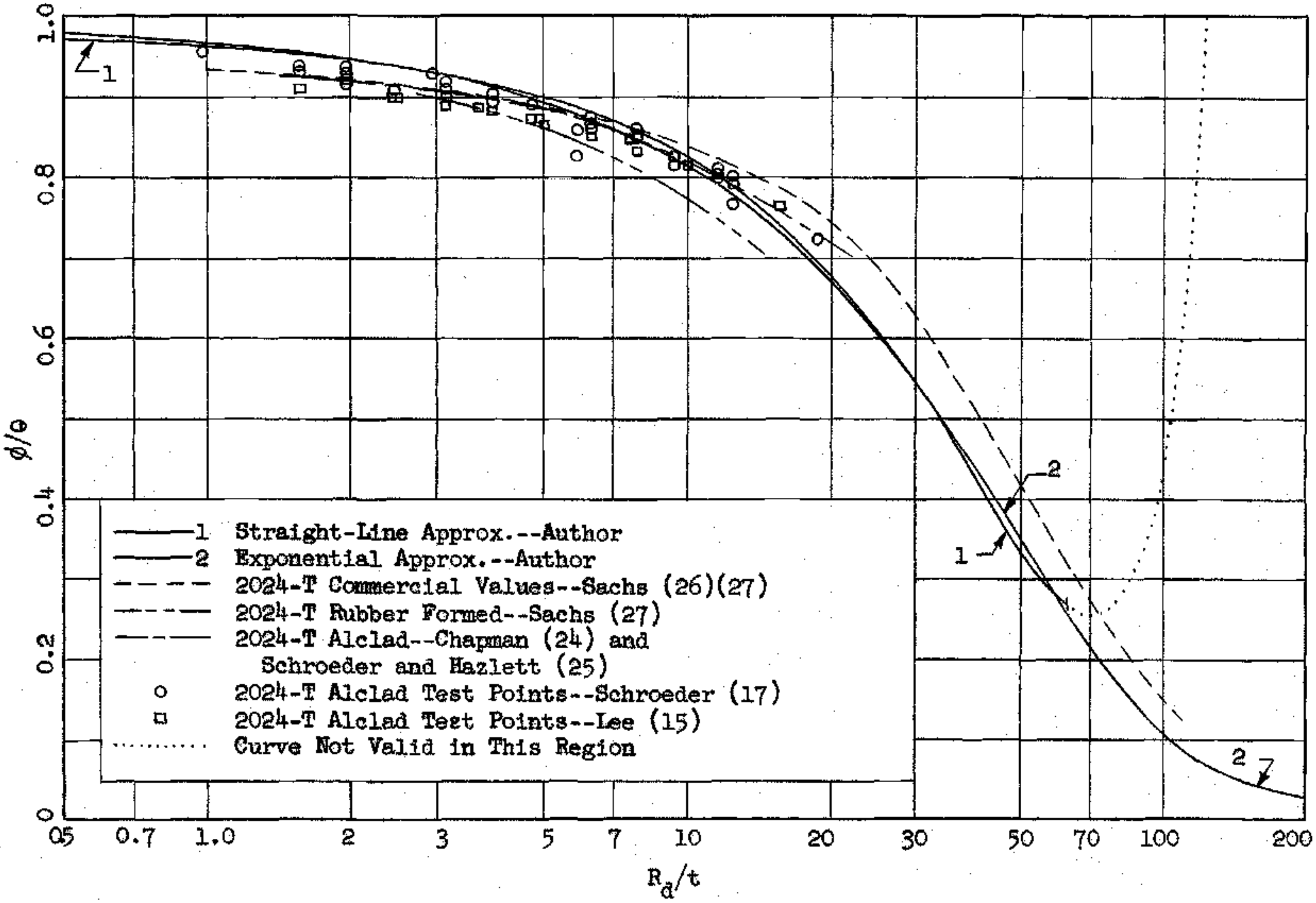

Figure 13. Springback Ratio for 2024-T--Theoretical Results and Test Data

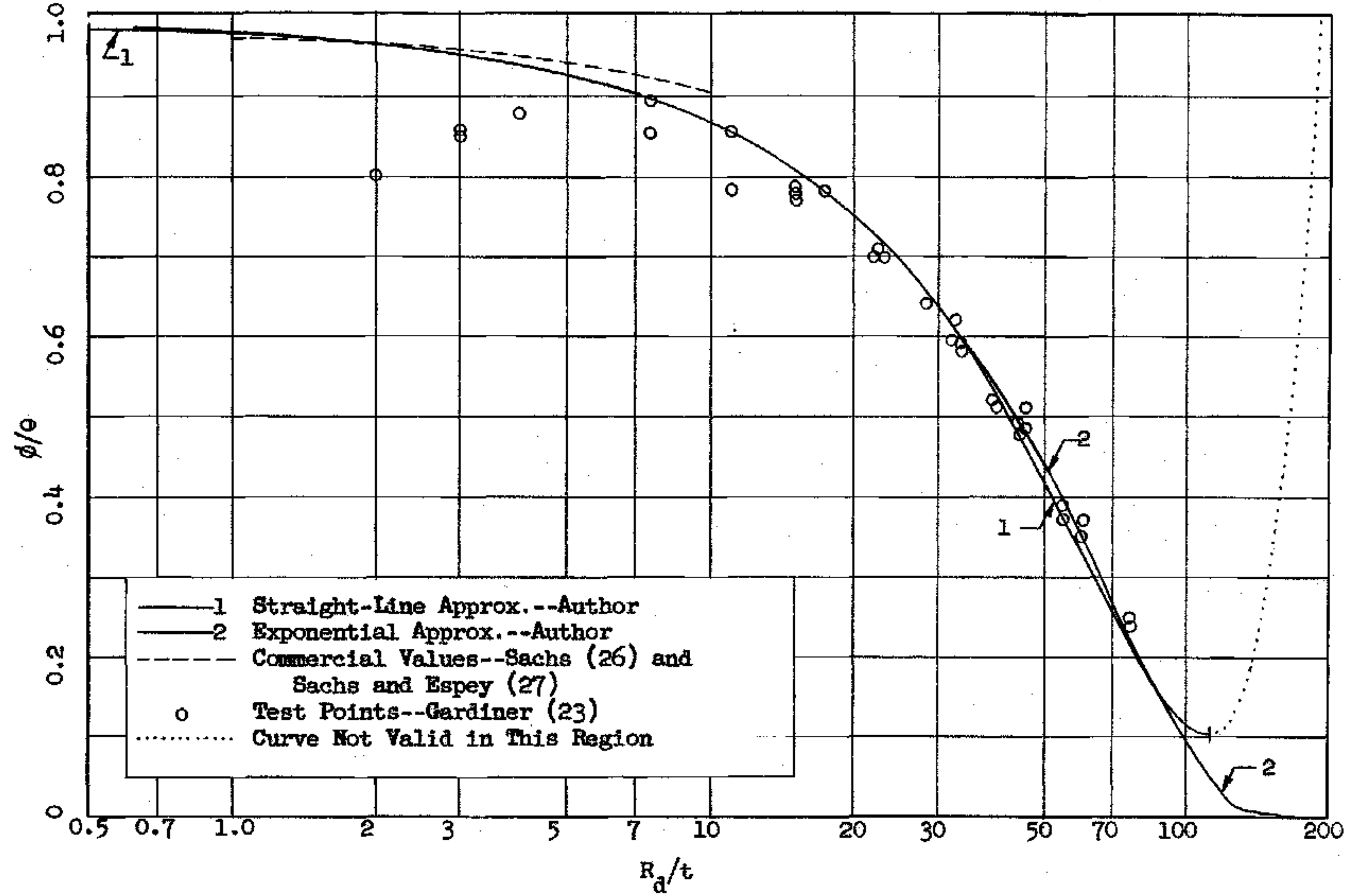

Figure 14. Springback Ratio for 6061-T6--Theoretical Results and Test Data

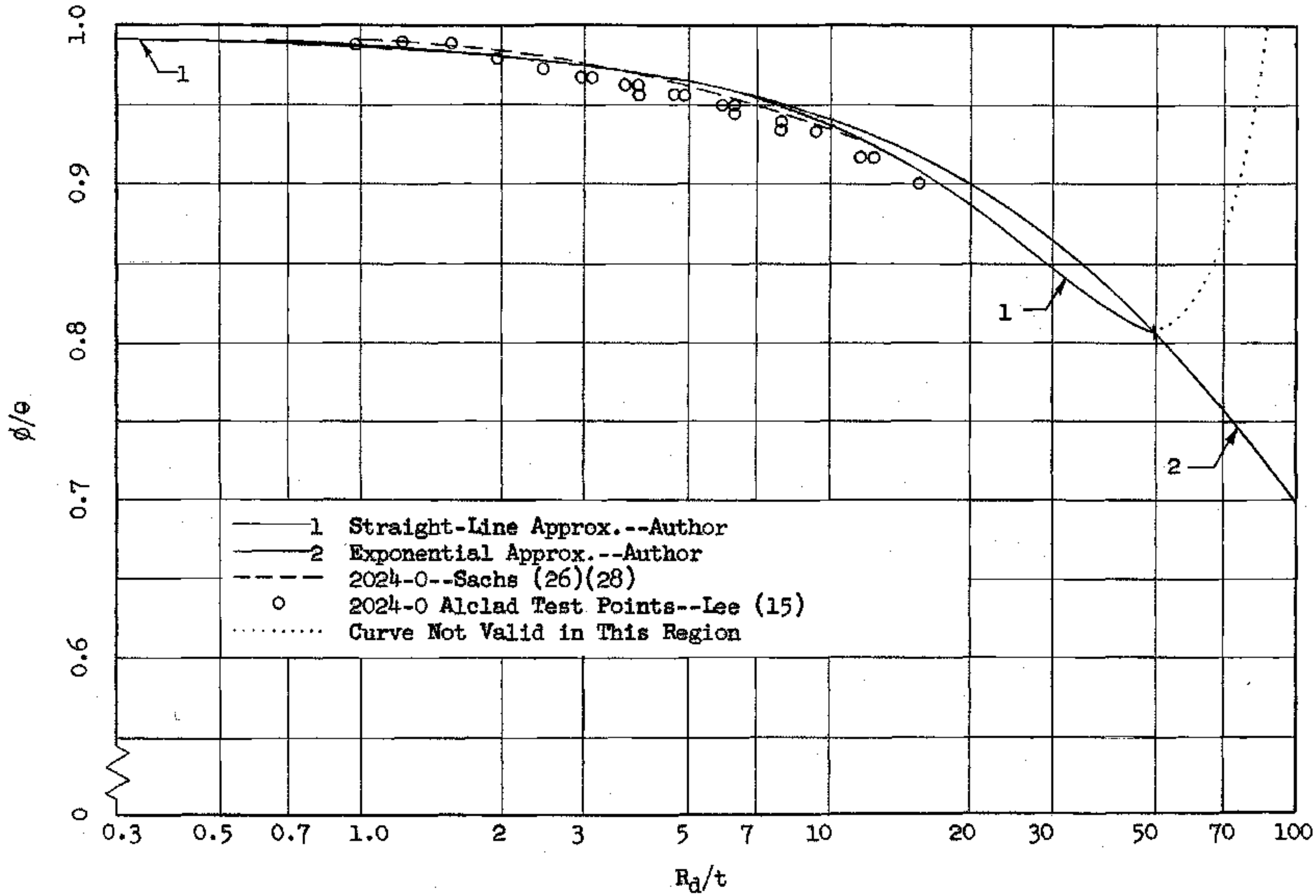

Figure 15. Springback Ratio for 2024-0--Theoretical Results and Test Data

Theory Presented by Schroeder (22).--Assuming that the neutral axis of springback passes through the centroid, Schroeder derived the relation

$$R_p = \frac{R_d}{1 + \frac{q_x R_d}{1 + e_2}} \tag{70}$$

where e_2 here denotes the bending strain of the innermost fiber of the member and q_x is a parameter. He also assumed a stress-strain curve of the form of Figure 6(e), equal in tension and compression, where $s_o = s_y$. Under this assumption, for the case of pure bending of a cross-section symmetrical about an axis normal to the plane of bending, the neutral axis in bending passes through the centroid. Hence, for a rectangular cross-section one may obtain the expressions

$$1 + e_2 = \frac{R_d}{R_d + t/2} \tag{71}$$

and

$$\frac{\phi}{\theta} = \frac{R_d + t/2}{R_p + t/2} \tag{72}$$

Using Schroeder's method, for the case under study one obtains

$$q_x = - \frac{3s_y}{Et} - \frac{E'}{ER_d} \tag{73}$$

Combining the above expressions yields the relation

$$\frac{\phi}{\theta} = 1 - \frac{\frac{3s_y R_d}{E't} + 1}{\frac{E}{E'} - \frac{3s_y}{2E'} - \frac{t}{2R_d}} \tag{74}$$

In the calculations made here, E' was chosen as follows:

2024-T; E' = 135 ksi

6061-T6; E' = 40 ksi

2024-0; E' = 160 ksi

<u>Theory Presented by Gardiner (23)</u>.--Gardiner assumed that the neutral axis always passes through the centroid of the member and that the stress-strain curve is of the form of Figure 6(d), equal in tension and compression, with $s_o = s_y$. He then derived the relation

$$\frac{\phi}{\theta} = 4\left[\frac{R_d s_y}{Et}\right]^3 - 3\left[\frac{R_d s_y}{Et}\right] + 1 \tag{75}$$

Table 12. Calculated Values--Theories of Schroeder (22) and Gardiner (23)--2024-T

R_d/t (in./in.)	ϕ/θ (Schroeder)	ϕ/θ (Gardiner)
0.100	0.9833	0.9984
0.15	0.9841	0.9977
0.20	0.9835	0.9969
0.25	0.9828	0.9961
0.30	0.9820	0.9954
0.40	0.9806	0.9938
0.50	0.9791	0.9923
0.60	0.9776	0.9907
0.70	0.9750	0.9892
0.80	0.9745	0.9876
0.90	0.9729	0.9861
1.00	0.9714	0.9845
1.5	0.9636	0.9768
2.0	0.9558	0.9690
2.5	0.9470	0.9613
3.0	0.9402	0.9536
4.0	0.9246	0.9381
5.0	0.9100	0.9227
6.0	0.8934	0.9072
7.0	0.8778	0.8918
8.0	0.8622	0.8764
9.0	0.8466	0.8611
10.0	0.8310	0.8457
15	0.7530	0.7696
20	0.6750	0.6948
25	0.5960	0.6216
30	0.5190	0.5504
40	0.3630	0.4159
50	0.2070	0.2946
60	0.0510	0.1899
70	-0.1051	0.1048
80	-0.2483	0.0429
90	-0.4171	0.0074
100	-0.5731	0.0016
150	-1.3532	0.5330
200	-2.1333	2.3012
250	-2.9006	5.7184
300	-3.6807	11.1968
400	-5.2537	29.9866
500	-6.8139	61.9686

Table 13. Calculated Values--Theories of Schroeder (22) and Gardiner (23)--6061-T6

R_d/t (in./in.)	Ø/Θ Schroeder)	Ø/Θ (Gardiner)
0.100	0.9947	0.9988
0.15	0.9941	0.9982
0.20	0.9935	0.9976
0.25	0.9929	0.9970
0.30	0.9924	0.9964
0.40	0.9912	0.9953
0.50	0.9900	0.9941
0.60	0.9888	0.9929
0.70	0.9876	0.9917
0.80	0.9863	0.9905
0.90	0.9852	0.9893
1.00	0.9830	0.9882
1.5	0.9781	0.9822
2.0	0.9721	0.9763
2.5	0.9662	0.9704
3.0	0.9602	0.9644
4.0	0.9483	0.9526
5.0	0.9364	0.9408
6.0	0.9244	0.9290
7.0	0.9125	0.9171
8.0	0.9006	0.9053
9.0	0.8887	0.8935
10.0	0.8767	0.8817
15	0.8171	0.8231
20	0.7414	0.7650
25	0.6979	0.7076
30	0.6383	0.6512
40	0.5191	0.5418
50	0.3999	0.4383
60	0.2807	0.3422
70	0.1615	0.2550
80	0.0423	0.1782
90	-0.0769	0.1132
100	-0.1961	0.0615
150	-0.7921	0.0545
200	-1.3882	0.6022
250	-1.9842	1.8894
300	-2.5802	4.1010
400	-3.7723	12.0373
500	-4.9644	25.890

Table 14. Calculated Values--Theories of Schroeder (22) and Gardiner (23)--2024-0

R_d/t (in./in.)	ϕ/θ (Schroeder)	ϕ/θ (Gardiner)
0.100	0.9834	0.9997
0.15	0.9837	0.9996
0.20	0.9838	0.9995
0.25	0.9838	0.9994
0.30	0.9837	0.9992
0.40	0.9836	0.9990
0.50	0.9834	0.9987
0.60	0.9831	0.9985
0.70	0.9829	0.9982
0.80	0.9827	0.9980
0.90	0.9824	0.9977
1.00	0.9822	0.9974
1.5	0.9810	0.9962
2.0	0.9797	0.9949
2.5	0.9784	0.9936
3.0	0.9772	0.9924
4.0	0.9746	0.9898
5.0	0.9721	0.9873
6.0	0.9696	0.9847
7.0	0.9670	0.9822
8.0	0.9644	0.9796
9.0	0.9620	0.9771
10.0	0.9534	0.9745
15	0.9466	0.9618
20	0.9338	0.9491
25	0.9211	0.9364
30	0.9084	0.9236
40	0.8828	0.8983
50	0.8573	0.8729
60	0.8318	0.8477
70	0.8063	0.8225
80	0.7808	0.7975
90	0.7553	0.7725
100	0.7298	0.7477
150	0.6023	0.6262
200	0.4748	0.5102
250	0.3473	0.4015
300	0.2197	0.3020
400	-0.0353	0.1378
500	-0.2903	0.0324

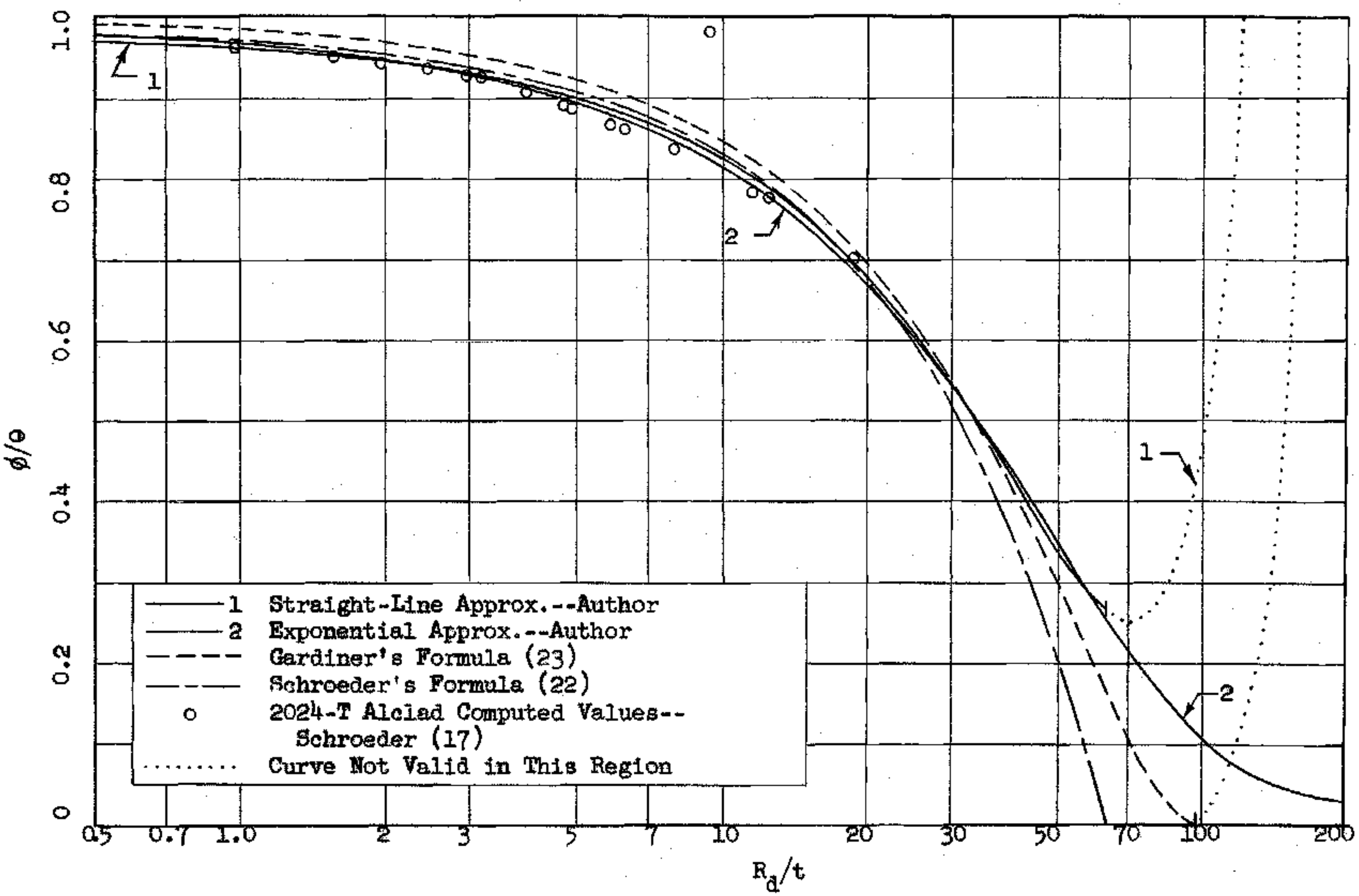

Figure 16. Springback Ratio for 2024-T--Comparison of Theories

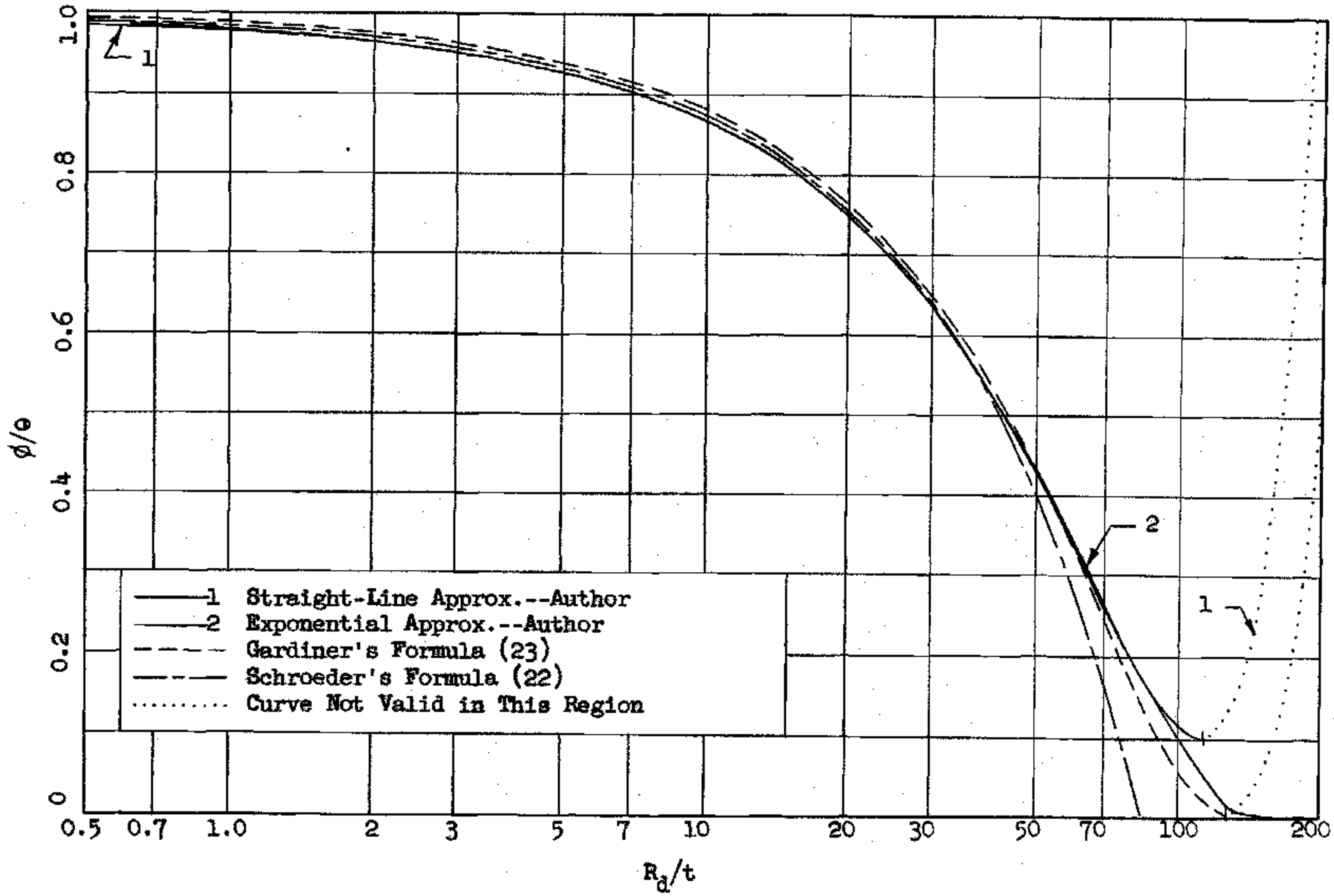

Figure 17. Springback Ratio for 6061-T6--Comparison of Theories

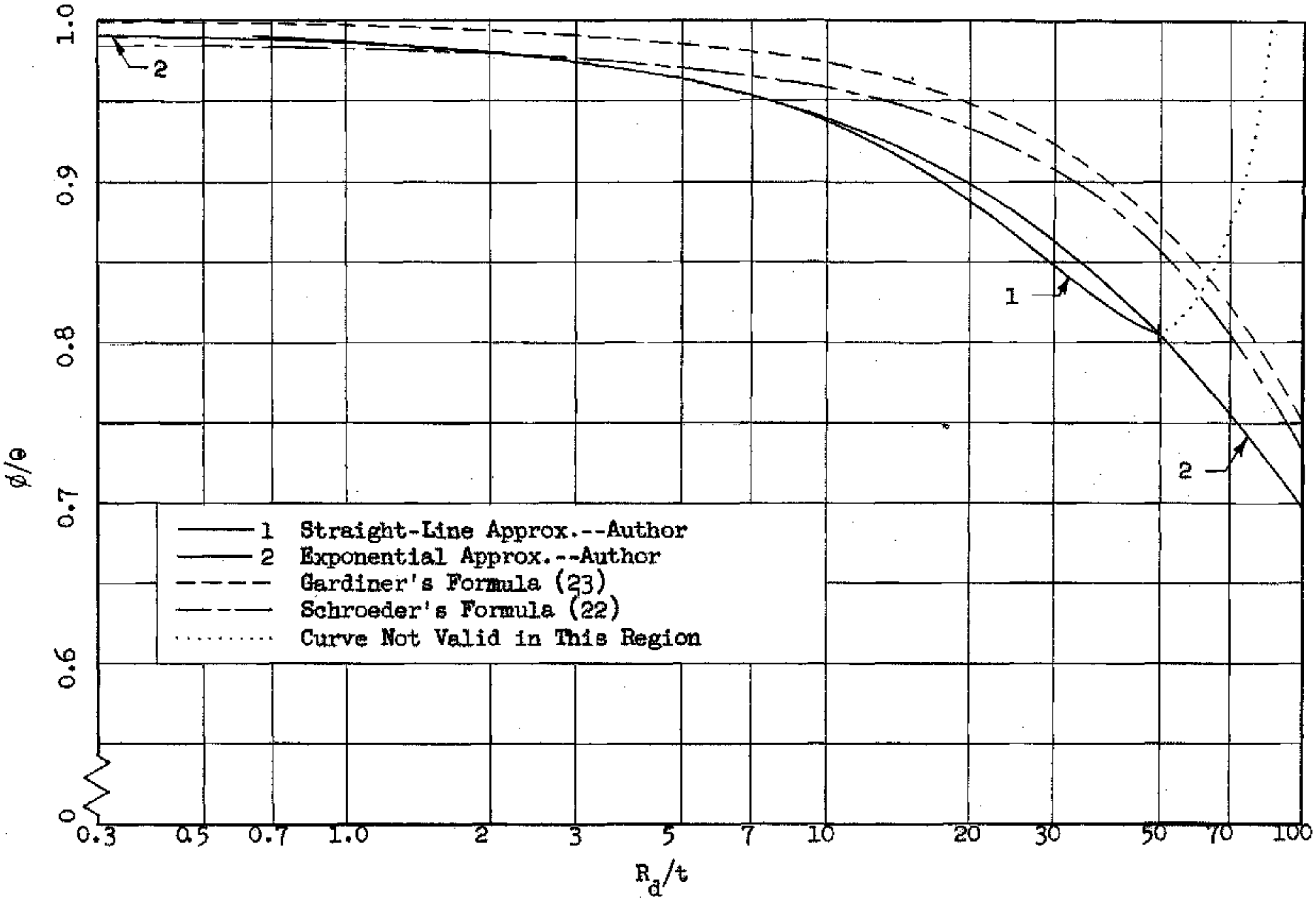

Figure 18. Springback Ratio for 2024-0--Comparison of Theories

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